Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 209 JUNE, 1930 No. 6

# RESEARCH IN THE CELLULOSE INDUSTRIES.*

BY

RALPH H. McKEE, Ph.D., LL.D.,

Professor of Chemical Engineering, Columbia University.

In the past, science and industry were distinctly separate fields of activity and each looked askance at the other. The research worker was engaged in investigating and interpreting obscure phenomena with no thought for the industrial applications of the discoveries he made. The "master of industry" had grown up in the business and on his retirement or death was succeeded by a son similarly trained. They called themselves practical men. You recall Disraeli's witty and truthful definition of practical men "as those who continue to practice the mistakes of their predecessors." To such the scientist was a musty dreamer.

Meanwhile, industry went its own sweet way, manufacturing and selling wherever there was a profit. As the industries grew older, it was necessary to search for other raw materials when local supplies were exhausted. From time to time with increase of competition the industries sought assistance and advice from the scientists to improve products, eliminate waste, and obtain new raw materials. The way to such results lay through *Research*.

What is research? Research is a systematic investigation to discover facts, to interpret these facts and to apply this interpretation to the problem in hand.

* Presented at the Stated Meeting held Wednesday, March 19, 1930.

Physicists and chemists consider their research a service necessary for the advancement of civilization. Chemical engineers, of whom I am one, agree with this, but consider it necessary that research shall, in addition, be profitable to those whom it serves.

Science is a pilot of industry. The principal advancements in science in recent years have been made possible only by concerted action. In the future this will be true in even greater measure for already the knowledge of any one man hardly suffices to cover all the fields involved in the solution of a single research problem.

The methods of scientific research are not new but their systematic use in industry is new. As yet, research is established in comparatively few industries. Industry will never appreciate the full value of research nor receive commensurate financial returns from it until it is practiced consistently and continuously.

The methods of industrial research can be applied with advantage to almost every kind of organization, large or small; whether for manufacture, transportation, or merchandizing. Tonight, however, I shall confine the discussion to research in industries based on cellulose.

These industries have particular importance for Philadelphia and Pennsylvania. The pulp and paper mills of the state and the large textile interests of Philadelphia and its vicinity are all dependent upon forms of cellulose as the principal raw material. Thus they are cellulose industries in the narrowest sense.

From all sides we hear that this is an Age of Cellulose. Indeed, no other substance enters into our life in so many ways. We use it as wood for housing, as cotton and rayon for clothing, and as paper to convey knowledge. In Europe sugars are made from sawdust to feed hogs and cattle, and alcohol—useful there as a beverage—is a byproduct in large quantity of paper pulp manufacture. Deprived of other materials, man could well-nigh subsist on cellulose and its products.

Industrial research on cellulose is not limited to testing incoming raw materials and the quality of outgoing products in comparison with competitors' goods. More attention is

given to devising methods for control of processes and operations in order to maintain maximum production at lowest unit cost and highest quality of the product.

In the narrower, but perhaps highest, sense research strives to improve present processes and products, to develop

The straight fibers are ordinary jute. The two groups of fibers in the middle are artificial wool. The fourth group of fibers is real wool, part black and part white.

new products by utilizing waste and by studying new raw materials.

A statistical study of the money spent for industrial research by 800 organizations showed an average of about 1 per cent. yearly on the capital investment.[1] Of this 1 per cent., 34 per cent. (in round numbers) is allotted to improve the quality of the products and service of products to consumers; 31 per cent. to improve manufacturing methods and thereby reduce cost; 20 per cent. to develop new uses for old products; and 15 per cent. to develop new products both from waste and from new raw materials.

In my opinion it would be more logical to base expenditure for research on the total sales value than on invested capital.

[1] National Research Council, "Research, a paying investment," New York, 1928.

Thus the budget item for research might be 1 per cent. of sales return. The fluctuation in sales would then be a direct index of the effectiveness of research and the allowance for it gaged accordingly.

Tonight I shall show you the results of three investigations in the cellulose field. The first is concerned with making a textile product more useful to the consumer; the second with making an artificial wool to be used either alone or mixed with wool from the sheep. Most of my time, however, will be given to a development which promises to be of aid in bringing back to such states as New York and Pennsylvania the wood using industries which they have been and now are losing. The method of accomplishing this is to develop fast-growing varieties of trees whose wood is particularly suitable for pulpwood and lighter types of lumber.

Several years ago The Rossendale-Reddaway Co., a New Jersey concern manufacturing cotton belting, found that one of their competitors had developed a belt which showed on test that it had about 6 per cent. longer life than their own product. They could not understand this difference because the competitor's belt seemed to be woven from the same kind of cotton yarn and to have the same general characteristics as their own.

Cotton belting is a web which looks like heavy canvas. It is treated with a dressing for the purpose of preserving it, lubricating the fibers, and causing the belt to adhere to the pulley. Belt dressings are the highly prized trade secrets of the manufacturers. The formulas are very complex and sometimes contain as many as twenty ingredients.

The New Jersey manufacturer did not know how to increase the life of his belt so he came to me as he had heard of my work in the cellulose field.

A study of the dressing showed that several constituents were undesirable. For example, some were organic acids or would produce organic acids when the belt was heated on the pulleys. Final solution of the problem resulted in a dressing containing but two materials: one a lubricant, the other a sticky adhesive. A slight change was made in the method of weaving. When the new belt was tested it had 200 per cent. advantage over that of the competitor. Thus it became

possible to make a belt 250 per cent. better than the original at a reduced cost.

The largest use for cotton belts is for the transfer of power in manufacturing plants. In such cases the belts are long and wide. I have here two small belts, made by The Rossendale-Reddaway Co., of a size that is used in enormous quantity as fan belts for automobiles. You may be interested in examining them later.

The prices of farm crops vary from season to season and even within a single season. This fluctuation is a heavy handicap to industries which depend on farm crops for raw materials.

Wool comes on the market twice a year, the American crop in June and the Australian crop six months later. Most fabrics require mixtures of various types of wool. Accordingly, the mills must purchase their year's supply of a particular grade of wool when it is on the market. This procedure not only ties up enormous sums of money in inventories but it also limits the amount of a particular type of fabric that can be made. If the price of wool drops, the mill will sustain a loss due to the decrease in value of the material on hand. For example, the last annual report of one of our American manufacturers of wool fabrics shows a loss of more than $20,000,000 due to the decreased value of the wool on hand December 31, 1929, below the price which was paid for that wool earlier in the year.

The advantage of a synthetic or artificial textile fiber is that it can be produced during the year according to demand and that the quality will not vary from month to month. In addition, the consumer knows he can obtain whatever amount he needs and that it will be of uniform quality. Neither the manufacturer nor the textile mill needs to carry a large inventory. Furthermore, such products are comparatively stable in price throughout the year. Inventory write-offs become unimportant. The artificial silks, some of which go by the name of rayon, belong to this class and are being welcomed into the textile industry partly because of the situation which I have just mentioned.

Many attempts have been made to produce an artificial wool which could be used in place of, or mixed with, natural

wool. Of these, two types are outstanding: one based on rayon, the other on vegetable fibers.

Artificial silk, or rayon, spun for this purpose is heavier than ordinary rayon. Numerous means have been devised to impart a crinkle. On account of the elaborate method of manufacture, the price of this product is nearly as high as that of medium grade wools. It has the further disadvantage that the fiber is smooth and slippery so it fails to make a strong thread when mixed with wool. Indeed, it is often possible to pull out such artificial silk fibers from the finished fabric.

The second type of artificial wool is made from vegetable fibers, such as China grass, jute, and so forth. The advantage of jute is that the final product can be dyed with wool in the mixture to practically the same color as the wool. This is possible to a limited extent with other cellulosic fibers but is easier with a jute derivative. Moreover, to treated jute we can give a curl or crinkle closely resembling that in natural wool. This is obtained by subjecting the jute to a strong caustic soda solution provided we do not keep it stretched during the treatment.

Four organizations in this country have made artificial wool from jute. The processes are all different but each has a lignin removing step, a bleaching step, and a crinkling step. The process which will dominate the field must, at a moderate cost, make fibers that are suitable to the textile trade. The important factors are reasonable crinkle, fair strength and light color. Here are samples of an artificial wool that has been developed in my laboratory at Columbia University.

I shall now show you slides that will enable you to compare the appearance of fibers of artificial wool with those of natural wool.

The slide contains fibers of original jute, the material from which ropes, twine and burlap bags are made. In the center are two samples of artificial wool, one dyed, the other undyed. Here is a sample of real wool. You can easily see the close resemblance of the artificial wool to the natural wool.

Such artificial wool fibers have been used in carpets and even in cloth for men's suits, but generally mixed with an equal amount of real wool.

Samples of cloth containing artificial wool which I have seen would be considered by the ordinary clothing merchant as of "all wool" composition.

An artificial wool acceptable for textile use will sell in clean, carded condition for about twenty cents a pound and the price would be the same year in and year out.

Individual fibers of jute (2 straight), artificial wool (2 undyed, 2 dyed), and real wool (1 black, 1 white).

Eighty years ago New York was the leading lumber producing state in the Union and Pennsylvania was a close second. Today these states have almost forgotten that they once supplied the lumber or pulpwood for their own mills. They use about twenty-five times as much as they produce. About three years ago a commission was created by the New York State legislature to make a survey of the wood industry of the state. The commission found that, not only many mills using timber directly, but other industries as well, have moved to locations nearer a source of wood supply. In fifteen years New York lost more than twelve hundred wood-using industrial plants.

States such as New York and Pennsylvania have four to

five million acres each of abandoned lands, lands which contribute nothing to the prosperity and welfare of the state and which have become a liability instead of an asset. Many of these abandoned farms and cutover areas can be purchased for from three to eight dollars an acre. To what better purpose can this area be devoted than to plant it with trees?

Growing trees appears to be a problem requiring generations rather than years or decades to give results. However, using modern methods, this need not be so.

*Wood Used in U. S. for Paper Making.*[1]

| Year. | Cords. |
|---|---|
| 1870 | 2,200 |
| 1880 | 41,000 |
| 1890 | 583,200 |
| 1900 | 1,986,310 |
| 1910 | 4,001,607 |
| 1920 | 6,114,072 |
| 1926 | 9,927,000 |

Paper consumed in U. S. in 1926 equivalent to 12,574,000 cords.

It is common knowledge that agricultural crops, such as wheat, oats, rice, corn, and so forth, have been improved by hybridization and selective breeding. Among tree crops hybridization has yielded conspicuous results in certain fruit and nut trees and in a few ornamental varieties. It is but logical to expect similar improvement in forest trees suitable for pulpwood and lumber if we applied the methods of hybridization.

Hybridization is, as you know, the cross-breeding of individuals of different species. The loganberry is a cross between two well-known species of berries, the antwerp raspberry and an ordinary blackberry. The mule is a cross between the jackass and the horse. Hybrids, in general, grow faster, are freer from disease, and in the case of animals eat less, than either of the parents. This is known to geneticists as "hybrid vigor." Unfortunately, hybrids do not breed true. For example, a tree grown from the seed from a hybrid will not be like the hybrid. Another disadvantage is that hybrids are often sterile.

[1] U. S. Department of Agriculture, Bulletin 1241, July 1924.

Hybridization must be carried out where the flowers are, i.e., high up in the tree. The trees are climbed (1) to protect female flowers from normal pollination, (2) to secure pollen, (3) to make the pollination, (4) to replace the paper sacks with green cheesecloth sacks, and (5) to collect ripe seeds.

By 1916 the wood-using industries were beginning to feel the shortage of their raw material. The price of wood per cord had tripled in the previous dozen years. In that year, 1916, in an address before the American Paper and Pulp Association, I said:

"The practical solution of the problem will be to get together specimens of the twenty-five or thirty varieties of poplar known in this country and abroad, cross them, grow the resulting hybrids, and test the woods obtained from these hybrids for pulpmaking qualities. From the results obtained, choose the hybrid that in growth of wood, quality of wood considered, is the best, and, using this tree as a source of stock, reforest the cutover pulpwood lands." [3]

At that time pulpwood averaged about ten dollars a cord delivered at the pulp mill. Attempts to get organizations interested in reforestation to support such experiments were not successful until about ten years later when the price of pulpwood had doubled. The Oxford Paper Company then agreed to finance the investigation and the work was done under my direction in coöperation with the New York Botanical Garden.

The solution of such a problem requires the concerted action of men of different training. The men whose aid was most important in this research were: Professor A. B. Sout, one of the leading geneticists in this country, of the New York Botanical Garden; Mr. E. J. Schreiner, a plant pathologist, Dr. D. E. Cable, a chemical engineer with exceptional knowledge of paper pulp; Mr. William Willets, a young man trained in paper technology and to whom most of the fiber studies are to be credited; Mr. Stephen Lawler, the field superintendent of the nursery in which the later work of propagation was carried on.[4]

In considering the improvement of trees for forest products the poplars seemed most promising. There were five hybrid poplars in existence so we knew that crosses could be obtained.

The Lombardy, which was an accidental hybrid about four hundred years ago in the north of Italy, is a male tree which

[3] McKee, Ralph H., "Proper Reforestation." Paper, February 23, 1916.

[4] Stout, A. B., McKee, Ralph H., and Schreiner, E. J., "The Breeding of Forest Trees for Pulp Wood," *Jour. of the N. Y. Botanical Garden*, 28, 49–63 (1927).

Cross-breeding of poplars is much simplified if one can work standing on top of a building. (Mr. E. J. Schreiner of Oxford Paper Company hybridizing at Geneva, New York.)

Sometimes one is fortunate in finding the desired female tree growing next a building. (Dr. A. B. Stout of the New York Botanical Garden.)

has been propagated by cuttings or slips. There has never been a female Lombardy poplar. Indeed, we can say that there has been only one Lombardy poplar tree, for the various trees are but parts of the original hybrid.

The second hybrid poplar is known as *P.* (*populus*) *regenerata* and was also an accidental cross. It occurred in a nursery near Paris in 1814.

The Carolina poplar, *P. Eugenei*, commonly incorrectly called in this country North Carolina poplar, is a hybrid. The first tree originated in 1833 at Metz, France, and was named for Prince Carol. It is still standing, and I understand, is now nearly thirty-five feet in circumference five feet from the ground.

In 1912, Professor Augustine Henry, of the Royal College of Science, Ireland, used trees at the Royal Botanical Gardens at Kew, England, and succeeded in artificially pollinating the female flowers of one poplar with the pollen from the male flowers of another. He obtained four seedlings, two of which were propagated under the name of *P. generosa*.

It may interest you to hear something of the methods we used in producing hybrid poplars. Our first problem was to locate the different varieties of poplars in this country. The largest collection was found to be at Highland Park in Rochester, the second largest at New York Botanical Garden. Other locations which furnished particular trees were Arnold Arboretum in Boston, and certain Long Island nurseries.

Before the flowers on the female trees opened, paper sacks were tied over the clusters to prevent normal pollination. Later, when the flowers opened, pollen was obtained from the flowers of the selected male tree. The pollen was taken at once to the female tree and, by means of a wad of cotton, dusted over the female flowers. The sacks were immediately replaced to prevent normal pollination. After about two weeks the paper sacks were replaced by green cheesecloth sacks. I might say that the only reason for staining the sacks green was to make them less conspicuous so that small boys would not throw stones at them.

Six weeks later when the seed had ripened, they were removed, taken to the laboratory where the cotton was carefully picked off and the seed planted in damp sand. Nature

A nursery row of Carolina poplars to be used the following spring as cuttings.

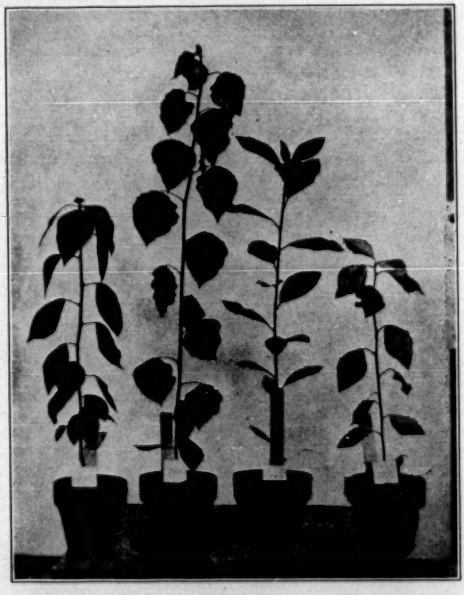
A nursery row whose hybrid parent stock was supplied by Governor Pinchot. To be used the following spring for cuttings.

Hybrid poplars just after the seeds have sprouted.

Some of the best of the hybrid poplar seedlings grown at the New York Botanical Garden. About 100 days old. When 16 months old these reached heights of 5 to 7 feet.

provides cotton so that the seed may be carried by the wind. Unfortunately this cotton is a handicap to growth and must be removed before the seeds are planted.

Poplar seeds need a mineral soil in order to germinate. In rich soils they do not germinate or if they do, the small

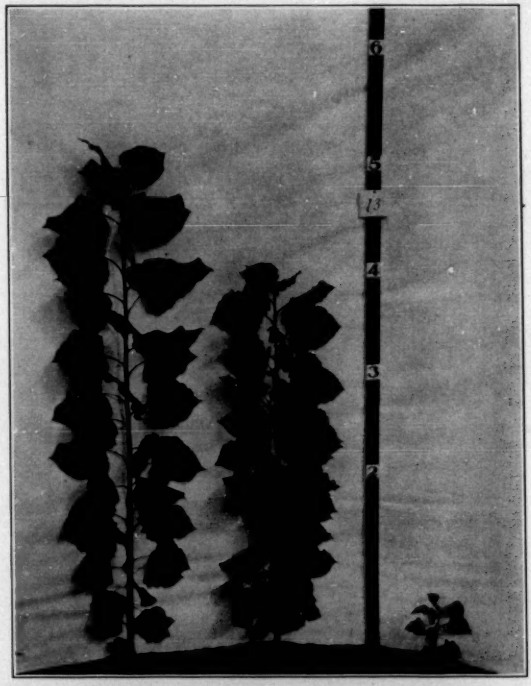


Some of the best hybrid poplars when 100 days from date of planting the seed. They are all in 3-inch pots. (New York Botanical Garden.)

plants soon die. When the seeds had germinated and the little trees had reached a height of about two inches, the baby trees were transferred to larger crocks and separated somewhat from each other.

When the seedlings were three to four inches high, each one was put into a separate pot and by the end of the growing season, about the middle of September, they were five to

eight inches in height. They were kept out of doors in cold frames through the winter and the next spring planted in the field nursery. By September hundreds of these had reached a height of over five feet, many six feet, and a few seven feet.

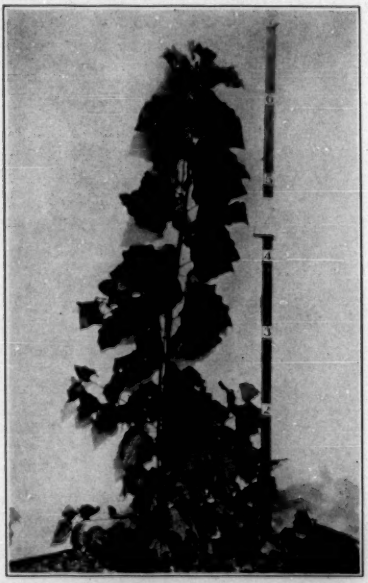



Three sister hybrid poplar seedlings. Note the variation in size though the seeds all came from the same catkin. One cannot tell beforehand which seed or which cross will give the most vigorous individuals.

A large number of the hybrids were, of course, no more vigorous than the parents, but fortunately many grew with greater vigor than either parent. The trial was exceptionally

successful. About fourteen thousand seedlings were grown and one hundred one new crosses were obtained.

There is little to indicate in advance of actual trial which crosses will yield offspring of noteworthy vigor. Rather more than one hundred of the best-growing hybrids were reserved



One of the best of the new hybrids. Good growth, nearly disease proof, propagates easily by cuttings.

and the remainder, nearly fourteen thousand, discarded. Each of these better seedlings was propagated by cutting it up into pieces ten or twelve inches long and stored for the winter. In the spring they were planted in the field nursery and by the next September each piece had become a tree six or seven feet tall.

Quick growth, however, was not the only factor to be

considered in the final choice of hybrids. The concern which is going to invest in a reforestation project requires a good yield per acre and wood of a quality that will suit both the lumber man and the pulp manufacturer. Therefore, in

Hybrid whose parents were *P. alba* and *P. adenopoda*. This is a fine tree but unfortunately it is not readily propagated by cuttings.

selecting hybrids for propagation from the hundred we chose the stocky trees as against the slender ones of the same height, for the stocky would produce the greatest yield per acre. The shape of the tree was also important. A tree

which is vertical and has few branches is to be preferred over one that is crooked and has heavy branches. Branches mean a distribution of the total wood in the tree and many knots—a source of trouble to the lumber man and pulp manufacturer alike.

In order to determine the quality of the wood, fiber studies were made and we found, as expected that the length of the

A fine tree from Mongolia but unfortunately winter kills it.

fiber varied in different hybrids. As longer fiber gives better paper and stronger lumber, we wanted to propagate the best quality possible. We were much pleased to find that fiber length and speed of growth were correlated. Thus, the fastest growing hybrids were those with the longest fiber.

Resistance to disease is also desirable and some of the hybrids were tested for common forest tree diseases. The results showed that many of the more vigorous hybrids were quite resistant to the more common poplar diseases.

The common wild poplars, *P. tremuloides* and *P. grandidentata*, from which much of the book and magazine paper is made, were compared with the hybrids. The studies disclosed

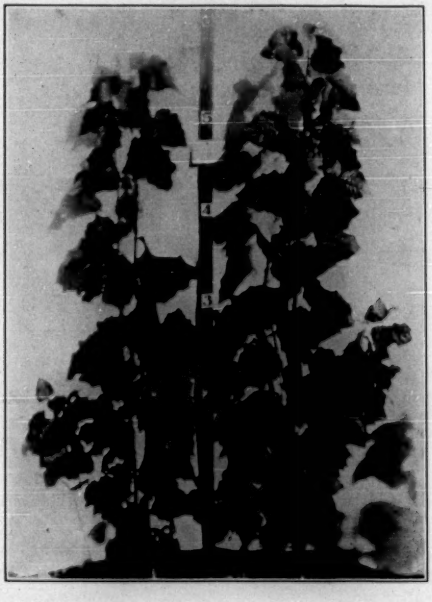
From a cross between two old world poplars. Though of vigorous growth, yet not satisfactory because it does not readily reproduce from cuttings.

that the speed of growth, that is, the weight of wood produced, of the new hybrids is from ten to fourteen times as much per year as the wild poplars growing under similar conditions.

After a tract of land is cut over it is customary to let the saplings that remain grow to trees. Such natural reforestation, as it is called, yields about six cords of useful wood per acre in sixty years, that is, an average of two hundred fifty pounds of pulpwood, or one hundred twenty-five pounds of

cellulose, per acre-year. For cotton, the yield per acre-year is one hundred and fifty pounds of cellulose, for flaxstraw—one hundred pounds, and for cornstalks, nearly five hundred pounds.

Well managed reforestation plantations of pulpwood using wild species will produce about two thousand pounds of



Good hybrid poplar seedlings about six feet high.

cellulose per acre-year. From the new hybrid poplar plantations we have every reason to expect eighty cords of pulpwood per acre in twelve years, that is, an average of about sixteen thousand pounds of merchantable wood per acre-year, equivalent to eight thousand pounds of cellulose per acre-year.

A handsome and fast-growing hybrid poplar but lacks one quality required to make it really useful: it is susceptible to a common poplar disease.

*Cellulose Production Per Acre-Year.*

| | |
|---|---|
| Flaxstraw | 100 lb. |
| Cotton | 150 lb. |
| Cornstalks | 500 lb. |
| Reforestation—Natural | 125 lb. |
| Reforestation—Managed | 2000 lb. |
| Reforestation—New Hybrid Poplars | 8000 lb. |

When trees can be grown so rapidly, they become an agricultural crop, but one without the uncertainties of the annual plants. A few of the outstanding reasons for using hybrid poplars for reforestation are that:

1. Wood is much more compact than forage crops. This is an advantage in handling, storage, and protection from decay, as well as in pulp mill operations.

2. Cut over hillsides are just as satisfactory for growing hybrid poplars as high priced farm land.

3. The tree can be bred to the best form both for yield and for cutting: a long, straight bole with few branches.

Cross section of *P. robusta* 23 years old and 17 inches in diameter. Note width of annular rings.

4. The large yield and short period of growth permit building of permanent roads and camps, as the cut over sections will be replanted.

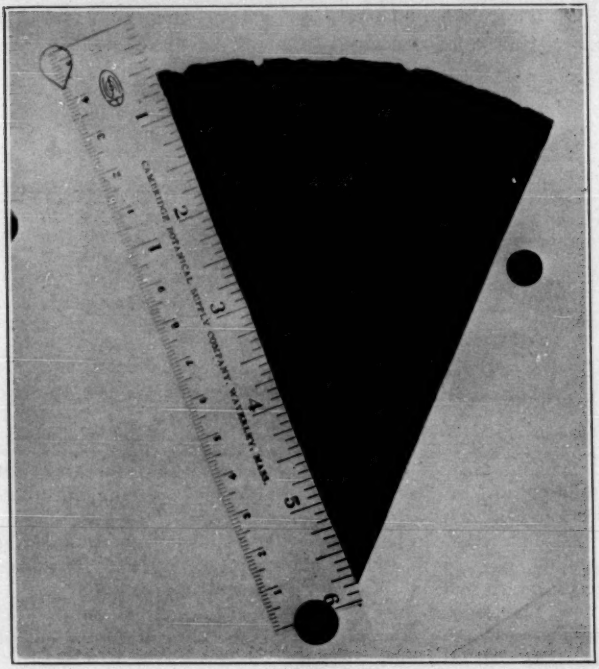
5. The trees can be left growing until wanted. Instead of spoiling, like a forage crop, they increase in size. It is not necessary to market the wood when the price is low.

6. The timber has a diversified market as lumber of various sorts and pulpwood which is the basis of some kinds of rayon as well as paper.

7. The investment risk on a woodpulp mill is very small as compared to that on a pulp mill using farm byproducts, for wood may be handled on a large scale, whereas straw and cornstalks are supplied to the pulp mill by many small growers.

8. Perhaps the most impressive, is that all calculations indicate that more dollars per acre-year of wood can be grown than dollars per acre-year of wheat or other common farm crops.

The whole value of reforestation is not complete with the reckoning of the dollar value of the wood grown. Such plantations protect the country from floods, from the erosion of arable land, and tend to preserve an even climate.

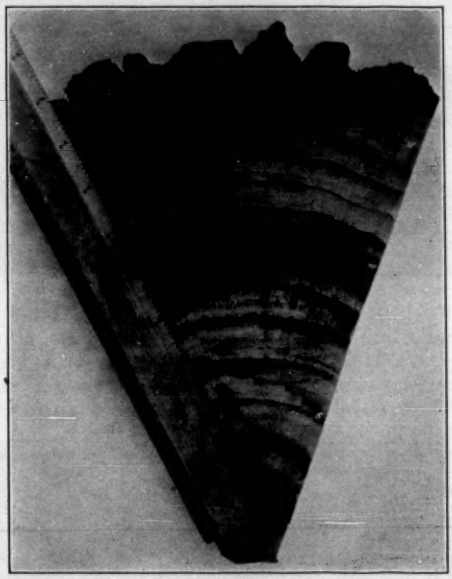



Note width of annular rings of common poplar (*P. tremuloides*).

Most of us take the enormous quantities of books and magazines for granted. It has been said that:

"There never was paper enough in the world until it was made of wood. But little more than 30 years ago an English authority hailed the development of the use of wood as

supplying an 'inexhaustible source' of useful fiber for paper making. Today we know that our resources are being drawn upon too heavily and that without better forest protection and more care for reproduction the pinch of



Note width of annular rings of a mediocre hybrid poplar. The best of the new poplars are expected to show rings of much greater width.

scarcity of paper-making materials will again be felt."[5] You have seen what research has accomplished to forestall such a condition. One company, the Oxford Paper Company, has already planted about two hundred acres with hybrid poplars for the use of its own pulp mill.

[5] Kellogg, R. S., "Pulpwood and Wood Pulp in North America," McGraw-Hill, 1923, p. 15.

Two photographs showing same man and same hybrid poplar in field plantation three weeks apart. Note increase of height of tree in three weeks.

## CONCLUSION.

At the beginning of the evening, we said that "in the past, science and industry were distinctly different fields of activity and each looked askance at the other." Industry as a whole no longer thinks of the scientist as a musty dreamer but rather as a powerful teammate. The feeling is by no means universal, however.

The three problems I have outlined to you show the increasing use of research. The belt dressing is an example of the practical man's first call to science for help in his problem. The artificial wool is the result of a combination of conditions: a plentiful raw material with certain well-known uses; a demand in the textile trade for a particular type of fiber; and adaptation of the one by research to meet the need of the other. The work with hybrid poplars is an attempt to develop a raw material which will relieve the increasing drain on natural supply. Of these three, one required weeks, another, months, and the third, years. All together they are but an infinitesimal portion of the work that has been and has to be done on cellulose.

In the case of such a complex problem as reforestation a single company can make but slight headway. The work must be supported by many organizations in all parts of the country. For example, a dozen organizations might engage in hybrid poplar propagation and use without interfering in the least with one another.

I hope that in this discussion you have caught a glimpse of the position of research in industries which use cellulose as raw material. If, in addition, you have come to realize the need of expanding and consolidating that position, my evening has been well spent.

**$10,000 Prize for Achievement in Science.** The largest single monetary award in America for scientific accomplishment has been created by Popular Science Monthly, which, beginning this autumn, will confer an annual prize of $10,000 accompanied by a gold medal, upon the American citizen who has been responsible, during the preceding year, for the achievement in science of greatest potential value to the world.

The award will be bestowed under the auspices of the Popular Science Institute, a research organization maintained by the magazine, of which Prof. Collins P. Bliss, associate dean, New York University, is director. The Institute has enlisted the services of twenty-four leaders in American science to serve as a Committee of Award, whose task it will be to select the prize-winning effort.

The prize will be conferred for the first time in September, 1930, and the initial period of scientific accomplishment to be considered by the Committee of Award will be the twelve months ending June 30, 1930. All scientific workers, professional and amateur, academic and commercial, are eligible.

Among the distinguished men comprising the Committee of Award are Dr. Charles G. Abbot, secretary, Smithsonian Institution; Dr. George K. Burgess, director, United States Bureau of Standards; Dr. Harvey N. Davis, president, Stevens Institute of Technology; Dr. Arthur L. Day, director, Geophysical Laboratory, Carnegie Institution; Dr. E. E. Free, consulting engineer; Dr. Arthur D. Little, president, Arthur D. Little, Inc.; Dr. John C. Merriam, president, Carnegie Institution; Dr. Robert A. Millikan, chairman, executive council, California Institute of Technology; Dr. Elmer A. Sperry, chairman, board of directors, Sperry Gyroscope Co.; Dr. Samuel W. Stratton, president, Massachusetts Institute of Technology; Dr. Elihu Thomson, director, Thomson Laboratory of the General Electric Co., Lynn, Mass.; Dr. Edward R. Weidlein, director, Mellon Institute of Industrial Research; Dr. Willis R. Whitney, director of research, General Electric Co., Schenectady, N. Y., Orville Wright, co-inventor of the airplane and ten others.

# POLARIZATION OF SENSITIZED FLUORESCENCE.

BY

ALLAN C. G. MITCHELL, Ph.D.,

Fellow of the Bartol Research Foundation of the Franklin Institute.

## INTRODUCTION.

BARTOL RESEARCH FOUNDATION
Communication No. 46.

It has been shown by Wood and Ellet, Hanle, and others [1] that mercury vapor, when excited by polarized light from a water cooled quartz mercury arc, emits polarized resonance radiation ($\lambda$ 2537 Å.), with certain orientation of electric vector of exciting light, magnetic field and direction of observation. MacNair [2] obtained similar results with cadmium vapor when it was excited by the cadmium line 3261 Å. Analogous experiments by Pringsheim and Datta [1] on the D lines of sodium showed that the $D_1$ line is never polarized whereas the $D_2$ line is partially so. Finally, Hanle and Richter [3] investigated the polarization of the mercury lines $\lambda\lambda$ 4047, 4358, and 5461 Å., obtained by step-wise excitation of the mercury vapor in the presence of nitrogen, and found that these lines are polarized in different ways depending on the direction of the electric vector of the exciting light, the magnetic field, and the direction of observation.

For the cases of the polarization of the 2537 Å. line of mercury and of the 3261 Å. line of cadmium, classical considerations for the excitation of a bound electron by a polarized light wave account for all the observed results. Thus, if a light wave, having its electric vector lying in the $x$ direction, excites a mercury atom in the absence of a magnetic field and the resonance radiation is observed in the $z$ direction, then the resonance radiation will also be polarized in the same direction as that of the exciting light. The rotation of the plane of

[1] For a review of the subject see W. Hanle, *Ergebn. der exakt. Naturwiss.*, **4**, 214 (1925).

[2] W. A. MacNair, *Phys. Rev.*, **29**, 677 (1927).

[3] W. Hanle and E. F. Richter, *ZS. f. Phys.*, **54**, 811 (1929).

polarization by magnetic fields in certain directions is also explained by the classical theory. On the other hand, the polarization phenomena exhibited by the resonance radiation of sodium and by the mercury lines λλ 4046, 4358, and 5461 Å., cannot be explained by simple classical considerations. Their properties, however, are completely explained by the quantum theory as applied to the Zeeman effect components [3] going to make up the different levels of the atom which give rise to the line emitted. These considerations of the Zeeman levels also account for all the observed facts in the polarization phenomena of the lines 2537 Å. of mercury and 3261 Å. of cadmium. A further discussion of this theory will be given in the main body of this paper.

An analogous phenomenon is the emission of polarized light by mercury vapor excited by a beam of electrons. Experiments of this type were performed by Skinner [4] and Quarder.[5] They found that most of the mercury lines excited were polarized with their electric vectors parallel to the direction of the electron beam; some, however, were polarized in the opposite sense. The maximum polarization of all lines occurred at the excitation potential of the line in question, and decreased with increasing electron velocity. Skinner and Appleyard [6] explained these results in the following manner. They showed that a magnetic field in the direction of the electron stream had no effect on the polarization and further that the emitted radiation was polarized exactly as resonance radiation is polarized in such a magnetic field if the electric vector of the exciting light is assumed to be parallel to the electron stream. Considering the limiting case when the electron has just enough energy to excite the atom and therefore has no velocity after the collision, they assumed, with Ellet, Foote and Mohler [7] that the vector change of angular momentum of the atom is in a direction at right angles to the initial direction of the motion of the electron. This assumption can be generalized for any velocities within a few volts of the critical potential. It is thus seen that the component of angular momen-

[4] H. W. B. Skinner, *Proc. Roy. Soc.*, **112**, 642 (1926).
[5] B. Quarder, *ZS. f. Phys.*, **41**, 674 (1927).
[6] H. W. B. Skinner and E. T. S. Appleyard, *Proc. Roy. Soc.*, **117**, 224 (1927).
[7] Ellet, Foote, and Mohler, *Phys. Rev.*, **27**, 31 (1926).

tum parallel to the field cannot change as a result of the collision—hence $\Delta m = 0$ where $m$, as is customary, denotes the component of angular momentum in the direction of the field. The polarization of the most of the lines observed follows from this principle and the known Zeeman components of the lines in question.

### THEORETICAL DISCUSSION.

We shall investigate, in this paper, an extension of the above experiments to the problem of carrying over of polarization on collision between two atoms, *i.e.*, the polarization of sensitized fluorescence. Consider, in the ideal case, two atoms $A$ and $B$ (Fig. 1), both having a $^1S_0$ state as the ground level and a $^3P_1$ state as an excited level, and suppose both atoms to be in a magnetic field. Let us denote the magnetic quantum numbers of the two atoms in the lower state by $m_A$ and $m_B$ and in the upper state by $m_A'$ and $m_B'$ respectively. In the lower state we have $m_A = 0$, $m_B = 0$ and in the upper state, which will split into three levels in the magnetic field, we have $m_A' = 0, \pm 1$; $m_B' = 0, \pm 1$. Let the atom $A$ be excited by resonance radiation having the electric vector parallel to the magnetic field, then the $\pi$ component, the component corresponding to no charge of angular momentum parallel to the field, will be absorbed and the level $m_A' = 0$ will be excited. The component of angular momentum of the excited atom in the direction of the field will then be zero. Allow this excited atom to approach slowly the atom $B$, which is in its normal state ($m_B = 0$), having its component of angular momentum parallel to the field also zero. Suppose the excited state of $A$ to be higher than that of $B$ and that the energy difference between the two is $\Delta E$, so that energy can be transferred by collision of the second kind from $A$ to $B$, then $B$ will become excited and the energy difference $\Delta E$ will go into translational energy. Then in the limiting case, where $\Delta E \to 0$ and when no atomic fields arise during the collision to perturb the atom, we ask whether angular momentum will be conserved on collision. If this were true the atom $A$ would go from its excited state $m_A' = 0$ to its normal state $m_A = 0$ and hence $\Delta m_A = 0$. The atom $B$ would then be excited from its normal state $m_B = 0$ to an excited state with $m_B' = 0$ and hence $\Delta m_B = 0$,

and none of the other states ($m_B' = \pm 1$) would be excited, and the radiation emitted by the $B$ atom (sensitized fluorescence) would be polarized in a direction parallel to the magnetic field ($\Delta m_B = 0$).

In the practical case, of course, we do not have conditions as simple as stated above. In the first place it is quite possible that rather large atomic fields exist between atoms on collision. We know, however, from experiments on the depolarization of mercury resonance radiation by added gases,[8] that the depolarizing efficiency of most gases is rather low, anywhere from 17 per cent. (He) to 70 per cent. ($N_2$) of the collisions being effective,[9] so that this effect may not be of a very disturbing nature. In the second place the relative kinetic energy of the two atoms before and after collision might be considered as detrimental to the conservation of angular momentum. However, Skinner and Quarder observed polarization in cases where the kinetic energy of the electron was several volts higher than the critical potential for the line emitted.

## THE EXPERIMENT.

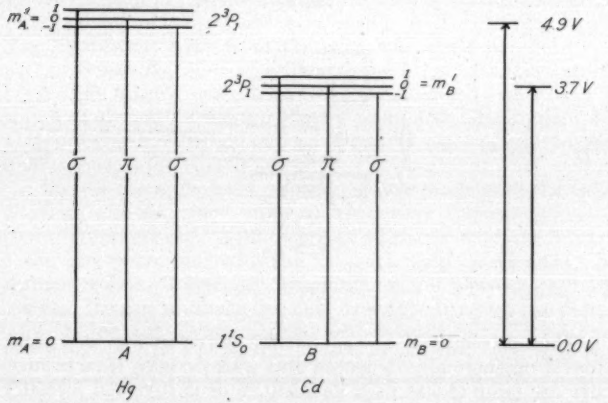
In this experiment we shall consider a collision of the second kind between an excited mercury atom in the $2^3P_1$ state and a normal cadmium atom ($1^1S_0$ state) to give an excited cadmium atom ($2^3P_1$ state) and a normal mercury ($1^1S_0$ state). Both atoms have the same type of Zeeman levels (Fig. 1). From the excitation potential of the two atoms (Hg = 4.9 v.; Cd = 3.7 v.) it will be seen that 1.2 volts can go into relative kinetic energy of the two atoms after collision. The only other atoms having similar Zeeman components but with an energy level lying very near 4.9 volts are the alkaline earths, but they have such low vapor pressures as to make the ex-

[8] V. v. Keussler, *Ann. der Physik*, **82**, 793 (1927).

[9] The very large depolarizing action of potassium and sodium on sodium resonance radiation is probably due to the large atomic fields associated with the unsymmetrical character of these atoms. (See for example, P. D. Foote, *Phys. Rev.*, **30**, 300 (1927).) These atoms have a magnetic moment associated with them which produces fields that become considerable at distances of the order of $10^{-8}$ cm. Since these fields are oriented at random with respect to the electric vector of the exciting light they will cause very great depolarization. The other case of large depolarization is that of mercury vapor on its own resonance radiation. This probably is due to secondary and tertiary radiation.

periment impracticable. Suppose now we radiate a mixture of mercury and cadmium vapors with polarized light of wave-length λ 2537 Å. from a mercury arc. Suppose the electric vector of the polarized light lies in the $x$ direction, and suppose the Cd-Hg mixture is in a strong magnetic

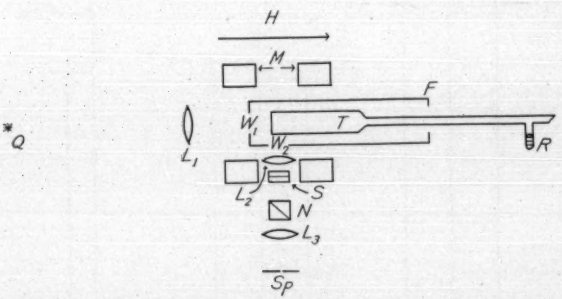
FIG. 1




field also in the $x$ direction, and we observe the resulting sensitized fluorescence along the $z$ axis. The mercury atom is excited from the normal state ($1^1S_0$, $m = 0$) to the $2^3P_1$ state ($m' = 0$) the $\pi$ component being excited, since the electric vector is parallel to the field. If it radiates, the radiation will be polarized ($\pi$ component) with the electric vector parallel to the field. If on the other hand it collides with a cadmium atom we ask whether angular momentum will be conserved and the cadmium atom be excited to a state and radiate light polarized with the electric vector parallel to the field. Again, suppose the electric vector of the exciting light is in the $y$ direction and is perpendicular to the field ($H = H_z$) and we observe along $z$, then the $\sigma$ components of the $2^3P_1$ state ($m' = \pm 1$) will be excited and the $\sigma$ components will be reradiated by the mercury atom in case it radiates before

making a collision, and the radiation will be polarized perpendicular to the field. If the excited mercury atom makes a collision with a cadmium atom before it radiates then we wish to know whether the $m' = \pm 1$ states of the cadmium atom rather than the state $m^1 = 0$ will be excited and whether the cadmium atom will radiate light polarized perpendicular to the field.

FIG. 2




The experimental arrangement is given in Fig. 2. Light from a magnetically deflected and water cooled quartz mercury arc lamp $Q$ was used as a source of unreversed mercury radiation of wave-length $\lambda$ 2537 Å. The light from the arc was sent through the lense $L_1$ and converged on the window of the quartz tube $T$. The diameter of the lense $L_1$ and its distance from the tube were such that the angular aperture of the beam was quite small. The quartz tube $T$, having a plane quartz window sealed to one end and with one side made flat near the window for observation, was connected to an ordinary high vacuum system. Cadmium was distilled into the tube and mercury vapor could diffuse in from a reservoir R, kept at room temperature. The gases helium or nitrogen could also be admitted to the tube at any desired pressure. The tube was lined up in the center of a pair of large Helmholtz coils $M$, capable of giving a uniform field of 300 gausses at the center. A furnace $F$ with quartz windows $W_1$ and $W_2$ could be placed around the tube as shown in the figure. This furnace was made of copper sheet wrapped with asbestos and con-

tained no iron so as not to distort the field; it was heated by two glowbars which produced practically no magnetic field in the furnace.

The optical arrangement is shown in the figure. No polarizer was used in the exciting beam, but since that angular aperture of the beam was small the electric vectors of the exciting light could be considered as essentially perpendicular to the direction of the beam and hence also to the field $H$ [10], thus causing a great gain in intensity of the exciting light. The fluorescent light from the tube was made parallel by a pair of lenses $L_2$, one of right and the other of left rotatory crystalline quartz, whose combined focal length was 7 cm.; it was sent through a Savart Plate $S$, a quartz Glans prism $N$, and converged on the slit of a small quartz spectrograph by the quartz lens $L_3$ (10 cm. focal length).

To test the apparatus, exposures were made with the tube $T$ filled with mercury vapor at a pressure corresponding to room temperature. With a correct adjustment of the Savart Plate and prism and with the magnetic field on, the 2537 Å. resonance line of mercury appeared on the plate crossed by distinct fringes, and was the only line appearing on the plate. This shows that we are dealing with resonance radiation and not light scattered from the surface of the tube and that the resonance radiation is polarized, in this case with the electric vector perpendicular to the field, in agreement with other experiments. Addition of 2 mm. of helium or 2 mm. of nitrogen caused the fringes of the 2537 line to lose their distinctness and to become diffuse, showing a decrease in the degree of polarization in agreement with the observatios of v. Keussler.[8] *The fringes, under these conditions, however, were still easily discernible on the plate. It is estimated that as little as 5% polarization could be detected with this apparatus.*

### EXPERIMENTS ON SENSITIZED FLUORESCENCE.

In most experiments on sensitized fluorescence a high pressure of mercury vapor is used, corresponding to temperatures between 60° C. and 100° C. and also a high pressure of cadmium vapor. This is done so that all the absorption of

[10] For a discussion of this method see Gaviola and Pringsheim, *ZS. f. Phys.*, **25**, 367 (1924); A. C. G. Mitchell, ibid., **49**, 228 (1928).

the 2537 Å. line of mercury occurs close to the front window of the tube, hence the sensitized fluorescence is concentrated near this window (Oberfläschenresonanz). The fluorescence is then usually observed from the front window, thus affording a great gain in intensity of light. In our polarization experiments, however, such high pressures of mercury are not allowable on account of its great depolarizing effect on its own resonance radiation. Further, since we wish to observe the fluorescence at right angles to the magnetic field we have to take the fluorescence off at right angles to the beam. For these two reasons low vapor pressures of mercury are necessary. All our experments are done, therefore, with mercury vapor at a pressure corresponding to its vapor pressure at room temperature. Cadmium was now distilled into the tube *T*, which had previously been baked out, and the whole system allowed to come to equilibrium at room temperature to insure the correct vapor pressure of mercury. The furnace was then heated to 400° C., but it was found that the cadmium distilled out of the tube carrying the mercury vapor with it, so that in half an hour there was no mercury vapor left in the tube. It was found that admission of 2 mm. of helium or nitrogen prevented this and that the mercury would than stay in the tube.

Experiments were then made with mercury vapor at about 0.001 mm. He at 2 mm. and cadmium at a vapor pressure corresponding to temperature between 350° and 450° C. All experiments were made with a magnetic field of 300 gausses in the direction shown in the figure.

### RESULTS.

With the furnace at 350° C. (vapor pressure of Cd about 0.5 mm) and 2 mm. of helium in the tube, a twelve hour exposure was taken. At the end of this time a ten minute exposure was taken on the same plate. Examination of the 12 hour exposure showed the mercury line 2537 Å. greatly overexposed, the cadmium line 3261 Å., and some of the scattered mercury lines. *The 3261 Å. line of cadmium showed no fringes across it, although they could easily have been detected on a line of its intensity had they been present.* The scattered mercury lines showed fringes, showing that they were polarized on reflection from the glass walls of the tube. The 2537 Å. line

was too overexposed to show any fringes. On the other hand, the ten minute exposure showed only the mercury resonance line 2537 Å. *The fringes could be easily seen on the line, showing that at the pressure of cadmium and helium in the tube the mercury resonance radiation was still distinctly polarized.* Other experiments confirm these results.

Another exposure was made in which the vapor pressure of cadmium was probably quite high, several millimeters, since a picture of the sensitized fluorescence was obtained in ten minutes, through the Savart Plate and the prism, showed the two lines Hg 2537 Å. and Cd 3261 Å., neither line exhibiting any fringes. This shows that when the vapor pressure of cadmium is high enough, cadmium vapor will depolarize mercury resonance radiation.

## DISCUSSION.

The fact that at a vapor pressure of cadmium of 0.5 mm. vapor pressure of mercury of 0.001 mm. and 2 mm. helium pressure, the cadmium 3261 Å. line shows no polarization while the 2537 Å. line of mercury does, leads to the conclusion that polarization is not carried over on collision.

It might be argued that the presence of 2 mm. of helium would give a number of metastable atoms ($2^3P_0$) and thus vitiate our conclusions. It has been shown however, in previous work that the number of metastable atoms formed at this low pressure of helium is exceedingly small and could not play much of a role in this experiment. One might also think that at 0.5 mm. pressure cadmium would depolarize its own resonance radiation in much the same way as high pressures of mercury depolarize the 2537 Å. line. This argument does not seem to be valid, since the depolarization of the 2537 Å. line of mercury by high mercury pressures is due to secondary and tertiary radiation, while in our case the cadmium 3261 Å. line exhibits a Doppler effect due to the fact that cadmium atoms have considerable kinetic energy (1.2 volts relative kinetic energy to the colliding mercury atom) and hence would not be absorbed to any considerable extent by the rest of the vapor.

It might be supposed that our argument concerning conservation of angular momentum on collision is not quite

correct, since the excited mercury atom would not in general always make a head-on collision with a cadmium atom. The problem of two atoms moving past each other would certainly give rise to some angular momentum (unquantised) but its order of magnitude would be about the same as the separation of rotational states in band spectra and would be quite small compared with the total electronic angular momentum with which we are dealing in this paper.

The result of this experiment shows that, under conditions where sensitized fluorescence can be observed, polarisation is not carried over on collision.

# ELECTRONIC CONDUCTION AND IONIZATION IN CROSSED ELECTRIC AND MAGNETIC FIELDS.

BY

NICHOLAS MINORSKY, Ph.D., E.E.,

Moore School of Electrical Engineering,
University of Pennsylvania.

## INTRODUCTION.

The action of a magnetic field on a discharge has been the object of many investigators. Comparatively simple is the case when the effect occurs in vacuo (i.e., pure electron discharge).[1, 2]

When gas is admitted into such a tube, the effect of the magnetic field becomes less marked, as if the gas had a tentency to maintain the current in spite of the action of the magnetic field.[3]

When the ionization of the gas sets in, the conditions become too complicated for a theoretical analysis to be attempted; there exists considerable literature relative to the effect of magnetic field on the discharge, especially in connection with the phenomena of rotation of discharges in radial fields.[4] When there is no ionization in the gas it seems possible to extend the original theory of Sir. J. J. Thomson in combination with the mechanism of elastic collisions between the primary electrons and the gas particles so as to explain the gradual drift of the electrons through the gas. This part

[1] J. J. Thomson, "Conduction of Electricity through Gases," Chapter IV, 1906.

[2] A. W. Hull, *Physical Review*, Vol. 18, 1921.

[3] O. W. Richardson and R. Chaudhuri, *Phil. Mag.* (6), **45**, 1923.

[4] A. de la Rive, *Arch. Se. phys. nat.*, t. 12 (1848), p. 222; *C. R.* (1849), p. 414; *Pogg. Ann.* (1858), p. 129; *Phil. Mag.* (4), t. 33 (1867), p. 512; H. A. Wilson and Martin, *Proc. Roy. Soc.*, t. 79 (1900), p. 417; D. N. Mallik, *Phil. Mag.* (6), t. 16 (1908), p. 531; A. Dufour, *Ann. de Chim. and Phys.* (8), t. 22 (1911), p. 282; C. E. Guye and A. Rothen, *Arch. Sc. Phys. nat.* (5), t. 3, p. 441; J. Nicol, *Proc. Roy. Soc., A*, t. 82 (1909), p. 29; H. Stolt, *Ann. der Phys.* (4), t. 74 (1924), p. 80; A. Hagenbach, "Handbuch der Physik" (1927), t. 14, Chap. VI, p. 337; N. Minorsky, *Journal de Physique*, April 1928.

of the problem is comparatively simple and is treated in the second section of this paper.

It is more difficult to obtain an expression for space charge conditions even in the simplest case of cylindrical symmetry which is assumed in the following. There seem to be two distinct methods of attack. The first method consists of introducing an indeterminate function representing the electronic density at a certain level, and of formulating the conditions of steady state at this level—this leads to an integral equation of the Fredholm type with a kernel representing the probability of transition between the electronic trajectories.[5] The difficulty of this method lies in the fact that the kernel of the equation is not known, since it depends upon the unknown mechanism of collisions between electrons and the gas particles. Its chief interest lies in the fact that it establishes the existence of a rigorous analytical solution on the basis of the classical theory.

The second method, developed in Section 3, is based on the assumption of a certain simplified mechanism of scattering in which instead of the double infinity of trajectories generated by collisions, one single trajectory is considered. To this average trajectory is attached a certain statistical weight resulting from the geometry of trochoidal motions. This imaginary distribution by discreet trajectories is, of course, very different from the actual distribution on a scale of the order of magnitude of trochoidal loops.[6] As regards the macroscopic scale (i.e., dimensions of the apparatus) presumably the change is very slight and it approaches zero when the size of the loops decreases.

Analytically the approximation consists of substituting for an integral equation a system of a finite number of linear equations with a suitable parameter.[7] Under those conditions it is possible to apply the binomial theorem of the theory of discontinuous probabilities and to gain thus an approximate

[5] This will form the subject of a later publication.

[6] In case of low voltage discharges (e.g., hot cathode tubes) and magnetic fields of the order 500–1000 gauss, the order of magnitude of trochoidal loops is roughly $10^{-2}$–$10^{-3}$ cm.

[7] R. Courant and D. Hilbert, "Methoden der Mathematischen Physik," Chapters I, II.

insight into the mechanism of the electronic propagation through an inert gas when the magnetic field is acting at a right angle to the electric force, and also to formulate the space charge conditions.

It must be noted that the mechanism of electronic conduction through the gas in crossed fields is essentially different from the case when the magnetic field is absent, since in the former the electrons move on the average at right angles to the electric force, whereas in the latter they move in its direction. One can also see this from the fact that the two phenomena do not tend to the same limit when the mean free path in the gas increases indefinitely. In fact, in vacuo, the electronic conduction, when only the electric force is present, is governed by the Langmuir's 3/2 power law, whereas with crossed fields it disappears entirely for certain critical values of the field.[2]

In cases where the size of trochoidal loops becomes considerable in comparison with the mean free path, the effect of magnetic field is less marked and the space charge effect caused by magnetic field disappears gradually for decreasing $H$.

In order to emphasize the characteristic feature of the phenomenon it will be assumed in the following that the length of one trochoidal loop is small in comparison with the mean free path.

GEOMETRY OF THE TROCHOIDAL SCATTERING OF ELECTRONS IN CROSSED FIELDS.

It is useful to simplify the theoretical investigation by introducing a number of assumptions under which the fundamental effect of scattering is particularly simple.[8]

It is always possible to waive one by one these assumptions and to study the more general cases. These assumptions are:

1. The gas is monatomic and without any appreciable electron affinity (e.g., noble gases, metallic vapors).
2. Collisions are elastic.
3. Velocities of thermal agitation are negligible.

[8] For the sake of simplicity the arrangement shown on Fig. 1 of the writer's article, published in *Journal de Physique* (Apr. 1928) will be assumed, with the exception that a circular filament is substituted for the pool of mercury. The emission in such a case occurs from each point of the abcissa axis.

4. The space charge effect is negligible (electric force is constant).
5. Initial velocities of the electrons are negligible.
6. The magnetic force is constant throughout the whole region.
7. The motion of the particles is studied in a plane tangent to the cylindrical surface (i.e., the radial motion of particles is neglected).

Of these assumptions, 1 and 2 can always be approximated sufficiently well by suitable experimental conditions; 3, 5, 6 and 7 are in general fulfilled sufficiently well. There remains only assumption 4, which generally is not fulfilled unless one works with very low magnetic fields and low electron emission. Unfortunately, there seems to be no method of proceeding initially without making this assumption, but the determination of the space charge constitutes the final object of the problem. The practical issue from this vicious circle is to admit assumption 4 and to proceed to determine the electron distribution to a first approximation; after the space charge conditions are thus determined it is necessary to start a second approximation in which for condition 4 there will be substituted the results from the first approximation.

The equations of motion of a charged particle in crossed fields are

$$\begin{aligned} m\frac{d^2x}{dt^2} &= He\frac{dy}{dt}, \\ m\frac{d^2y}{dt^2} &= -He\frac{dx}{dt} + eE. \end{aligned} \tag{1}$$

Putting $b = He/m$; $c = eE/m$ and transforming the variables $x$, $y$, into $\xi$, $\eta$, $\tau$ by means of the substitution

$$\begin{aligned} x &= b^{-2}c\xi, \\ y &= b^{-2}c\eta, \\ t &= b^{-1}\tau, \end{aligned} \tag{2}$$

equations (1) become:

$$\begin{aligned} \ddot{\xi} &= \dot{\eta} \\ \ddot{\eta} &= -\dot{\xi} + 1, \end{aligned} \tag{3}$$

where the dots designate the derivations with respect to $\tau$.

The substitution (2) simplifies considerably the subsequent calculations; it means that instead of dealing with the lengths $(x, y)$ measured in cm. and the time $(t)$ measured in seconds, the new coördinates of space $\xi = x/b^{-2}c$; $\eta = y/b^{-2}c$ and time $\tau = tb$ are expressed in terms of $b^{-2}c = E/kH^2 = a$ (i.e., radius of rolling circle of the cycloid given by (1) and $1/b = 1/kH$ (i.e., period of the electron in its orbit).

The integration of equations (3) with the initial conditions

$$\xi = \eta = \dot{\xi} = \dot{\eta} = 0 \text{ when } \tau = 0 \text{ gives } \xi = \tau - \text{Sin } \tau \text{ and } \eta = 1 - \text{Cos } \tau. \quad (4)$$

Equations (4) represent a cycloid.

A collision between a free electron and a molecule, occuring on this trajectory is characterized by two independent parameters: $\tau_0$ and $\phi$. The former $(\tau_0)$ indicates the point of the trajectory at which the collision occurs; the latter $(\phi)$ gives the angle between the incident and the reflected velocities of the electron. Assume, for example, that the collision occurs at the apex $(\tau_0 = \pi)$ of the trajectory and that it is of the "end on" type $(\phi = \pi)$. For the sake of convenience let such a collision be designated as of the $(\pi\pi)$ type (where the first $\pi$ means $\tau_0 = \pi$ and the second that $\phi = \pi$). A $(\pi\pi)$ collision gives rise to new initial conditions (taking a new origin of time $t = 0$ at the instant of collision) viz., when $\tau = 0, \xi = \pi, \eta = 2, \dot{\xi} = -2, \dot{\eta} = 0$, which give in conjunction with (3):

$$\xi = \pi - 3 \text{ Sin } \tau + \tau \text{ and } \eta = -3 \text{ Cos } \tau + 5. \quad (5)$$

The curve given by equations (5) is a trochoid having the same *rolling radius as the original cycloid (4) and a generating radius three times its rolling radius.* A following $\pi\pi$ collision on the trochoidal trajectory given by equation (5) gives rise to a third trochoidal trajectory having *the same rolling* radius and a generating radius equal to *five* times its rolling radius, and so on.

The following table contains data relative to the first four trajectories generated by the first three elastic $(\pi\pi)$ collisions. For the sake of convenience the variables $x$, $y$, $t$ are used instead of $\xi$, $\eta$, $\tau$ in this table. The constants of integration are not determined since only the velocity distribution as a function of $y$ is of interest here $(\omega = \kappa H)$.

| | $x$ | $y$ | $\frac{dx}{dt}$ | $\frac{dy}{dt}$ |
|---|---|---|---|---|
| First trajectory | $\frac{E}{H}t - a \operatorname{Sin} \omega t$ | $a - a \operatorname{Cos} \omega t$ | $\frac{E}{H}(1 - \operatorname{Cos} \omega t)$ | $\frac{E}{H} \operatorname{Sin} \omega t$ |
| Second trajectory | $\frac{E}{H}t - 3a \operatorname{Sin} \omega t$ | $5a - 3a \operatorname{Cos} \omega t$ | $\frac{E}{H}(1 - 3 \operatorname{Cos} \omega t)$ | $\frac{E}{H} 3 \operatorname{Sin} \omega t$ |
| Third trajectory | $\frac{E}{H}t - 5a \operatorname{Sin} \omega t$ | $13a - 5a \operatorname{Cos} \omega t$ | $\frac{E}{H}(1 - 5 \operatorname{Cos} \omega t)$ | $\frac{E}{H} 5 \operatorname{Sin} \omega t$ |
| Fourth trajectory | $\frac{E}{H}t - 7a \operatorname{Sin} \omega t$ | $25a - 7a \operatorname{Cos} \omega t$ | $\frac{E}{H}(1 - 7 \operatorname{Cos} \omega t)$ | $\frac{E}{H} 7 \operatorname{Sin} \omega t$ |

(6)

For the points $\omega t = \pi$, $dy/dt = 0$ and $dx/dt = E/H \times 2, 4, 6, 8 \ldots$ hence the kinetic energies of the electron passing through these points are in ratio $1:4:9:16:25:\ldots$ If, on the other hand, from (6) one forms the values for $y$ for the same points $\omega t = \pi$, one finds that these values are also in the same ratio: $1:4:9:16:25:\ldots$ Thus the kinetic energy increases in proportion to the distance through which the electron has fallen in the direction of $E$. This result could be, of course, anticipated since neither the magnetic field nor the elastic collisions contribute anything to the building up of the kinetic energy, and the latter is due exclusively to the electric field.

If the collisions occur at other points of the trochoidal trajectories (we still assume the central type of collisions for which $\phi = \pi$) the corresponding initial conditions must be taken into account in integrating equations (3). For example, a collision may occur at the lower part of the trochoid. For the lower apex it will be a $(\pi, 0)$ collision in our notation; it is easy to ascertain that its effect is opposite to that of a $(\pi, \pi)$ collision.

The preceding results, found for a very special type of the "end on" collisions, can be easily generalized. One finds (in $\xi, \eta, \tau$ system):

$$\begin{aligned} \xi &= \tau - \rho \operatorname{Sin} \tau, \\ \eta &= -\rho \operatorname{Cos} \tau + \frac{\rho^2 + 1}{2}, \end{aligned} \tag{7}$$

where $\rho$ is the generating radius.

Equations (7) represent a trochoid for a particular distance $\eta_0$ of the center of the trochoidal orbit from the cathode ($\eta_0 = \rho^2 + 1/2$). One verifies easily the results obtained in the particular case of the $(\pi, \pi)$ collisions (table). Equation (7) is thus quite general, resulting from the geometry of the trochoidal motion as well as from the assumption of elastic collisions.

Let $\dot{\xi}_1 + i\dot{\eta}_1$ be the velocity of the electron at the instant immediately preceding a collision expressed as a complex quantity in the $\xi$, $\eta$ plane, and let $\dot{\xi}_2 + i\dot{\eta}_2$ be that after the collision.

Since the collision is elastic immediately before and after the collision the velocity of the electron remains the same in magnitude but its direction changes abruptly by the angle $\phi$. Hence

$$\begin{aligned}\dot{\xi}_2 + i\dot{\eta}_2 &= (\dot{\xi}_1 + i\dot{\eta})\, e^{i\phi} \\ &= (\dot{\xi}_1 \operatorname{Cos}\phi - \dot{\eta}_1 \operatorname{Sin}\phi) + i(\dot{\xi}_1 \operatorname{Sin}\phi + \dot{\eta}_1 \operatorname{Cos}\phi) \qquad (8)\end{aligned}$$

and

$$\dot{\xi}_1^2 + \dot{\eta}_1^2 = \dot{\xi}_2^2 + \dot{\eta}_2^2. \qquad (9)$$

Designating by $\tau_0$ the instant of the collision, and noting that at the limit, $\xi_1 = \xi_0 = \xi_2$ and $\eta_1 = \eta_0 = \eta_2$, one has:

$$\begin{aligned}\xi_1 &= \tau_0 - \rho \operatorname{Sin}\tau_0 = \xi_2, \\ \xi_2 &= (1 - \rho \operatorname{Cos}\tau_0) \operatorname{Cos}\phi - \rho \operatorname{Sin}\tau_0 \operatorname{Sin}\phi, \\ \eta_1 &= -\rho \operatorname{Cos}\tau_0 = \eta_2, \\ \dot{\eta}_2 &= (1 - \rho \operatorname{Cos}\tau_0) \operatorname{Sin}\phi + \rho \operatorname{Sin}\tau_0 \operatorname{Cos}\phi.\end{aligned} \qquad (10)$$

The general integral of equations (3) is of the form:

$$\begin{aligned}\xi &= \pi + B \operatorname{Cos}\tau + C \operatorname{Sin}\tau + \tau, \\ \eta &= \quad - B \operatorname{Sin}\tau + C \operatorname{Cos}\tau + D,\end{aligned} \qquad (11)$$

where the constants of integration $A$, $B$, $C$, $D$ are determined by the initial conditions (10). This finally gives the parameters of the new trajectory as a function of the corresponding parameters $(\rho, \eta)$ of the old one and of the two additional parameters $(\tau_0\phi)$ determining the collision.

Omitting here the intermediate calculations one has finally:

$$\rho_2 = \sqrt{(\rho_1 \operatorname{Cos} \tau_0 - 1)^2 + \rho_1^2 \operatorname{Sin}^2 \tau_0 + 2\rho_1 \operatorname{Cos} (\tau_0 - \phi) - 2 \operatorname{Cos} \phi + 1}, \quad (12)$$

$$\eta_2 = D = \frac{\rho^2 + 1}{2} + 1 - \rho_1 \operatorname{Cos} \tau_0 + \rho_1 \operatorname{Sin} \tau_0 \operatorname{Sin} \phi + (\rho_1 \operatorname{Cos} \tau_0 - 1) \operatorname{Cos} \phi. \quad (13)$$

Equation (12) gives the generating radius $\rho_2$ of the new trajectory after the collision and (13) the ordinate $\eta_2$ of the center of rolling circle of this new trajectory, which is thus completely determined.

The preceding results for the particular case of the $(\pi, \pi)$ collisions (table (6)) are easily found from the general formulæ (12) (13) by putting $\tau_0 = \pi$ and $\phi = \pi$. There exists a double infinity of new trajectories $(\rho_2, \eta_2)$ corresponding to all possible values of the parameters $\tau_0, \phi$ determining a collision.

For a given trajectory $(\rho_1, \eta_1)$ the corresponding parameters $\rho_2, \eta_2$ of the new trajectory pass through maximum-minimum values when one has a straight "end on" apex collision (either $(\pi, \pi)$ or $(\pi, 0)$ in our notation).[9] For these extreme values of scattering there exist simple relations

$$(\rho_2) \text{ extr.} = \rho_1 \pm 2; \qquad (\eta_2) \text{ extr.} = \frac{\rho_1^2 + 1}{2} + 2(1 \pm \rho_1) \quad (14)$$

From the second equation (14) it follows that the centers of new trochoidal trajectories scattered by collisions cannot leave a band of the width $4\rho_1$ around the line of center of the original trajectory. This gives the zone within which the scattering can spread around the initial trajectory.

The above elementary analysis shows that the electronic motions occuring in an inert gas under the specified conditions are characterized by a trochoidal field of electronic trajectories having that property that the rolling radius $a = E/kH^2$ and the orbital period $T = 2\pi/kH$ are constants of the field depending only on $E$ and $H$.

Expressing the length and the time in a, $\tau$ units, the trochoidal field can be expressed in a single valued manner by means of equations (7).

[9] One finds this easily by equating to zero the partial derivatives of $\rho_2$ and $\eta_2$ with respect to the parameters $\tau_0, \phi$ determining the collision and by solving the equations thus obtained.

DISTRIBUTION OF ELECTRONS IN A TROCHOIDAL FIELD.

The preceding study must be supplemented by statistical considerations in order to be able to establish the conditions of distribution of electrons on various virtual trochoidal trajectories which is equivalent to the formulation of the space charge conditions to the first approximation.

In order to simplify the study let us assume that instead of a double infinity of scattered trajectories depending on two independent parameters $\tau_0$ and $\phi$ the scattering takes place by two discreet trajectories corresponding solely to either $(\pi\pi)$ or $(\pi o)$ types of collisions.

Furthermore, in order to introduce the probability considerations it is *assumed that for a given virtual trajectory (equation (7)) the probability of a $(\pi\pi)$ collision is proportional to the length of the upper arc of the trajectory and that of a $(\pi o)$ collision to the length of the lower arc of the same curve.*

This, in fact, is an a priori probability assumption which has no further justification,[10] and is equivalent to the *definition* of two statistical weights

$$\alpha_i = \frac{\sigma_i \text{ upper}}{\sigma 2\pi} \text{ and } \beta_i = \frac{\sigma_i \text{ lower}}{\sigma 2\pi}, \quad (\alpha_i + \beta_i = 1) \qquad (15)$$

to be attached to the $i$th trochoidal trajectory.

The calculation of the statistical weights $\alpha_i$, $\beta_i$ is thus reduced to that of the lengths of trochoidal tracks.

From the geometry of trochoidal curves one has:

$$d\sigma_i = \sqrt{1 + \rho_i^2 - 2\rho_i \operatorname{Cos} \tau}.d\tau$$

and

$$\sigma = (1 + \rho_i) \int_0^{\tau_1} \sqrt{1 - k_1^2 \operatorname{Cos}^2 \frac{\tau}{2}} \cdot d\tau$$

where

$$k_i^2 = \frac{4\rho_i}{(1 + \rho_i)^2}.$$

The upper limit $\tau_1$ corresponds to the point of the trochoid for which $d\eta/d\xi = \infty$ that is for $\operatorname{Cos} \tau_1 = 1/\rho$.

[10] In case of a roulette having red sectors two times wider than the black ones and for equal number of sectors the a priori probability to gain on red is $\frac{2}{3}$ and on black $\frac{1}{3}$. We can designate this particular probability as probability of the phase to distinguish it from the probability of collision introduced in the next section.

By known transformation the above integral is reduced to the elliptic integral of the second kind. The values of $\alpha_i$ and $\beta_i$ can be calculated in this manner.

It is possible now to apply the symbolic expansion of the product $\prod_{i=1}^{i=n} (\alpha_i S + \beta_i F)$ [11] in order to approximate the mechanism of electronic conduction in crossed fields.

In the above product $S$ designates "success," that is, in this problem the passage of an electron from a lower to a higher trajectory by means of a $(\pi\pi)$ collision and $F$ (failure) has a corresponding interpretation for a $(\pi 0)$ collision.

To complete our model of discreet trajectories it is preferable to consider that the electron emission occurs by abrupt steps separated by the periods $T = 2\pi/kH$ of no emission. Assume that at $\tau = 0$ the filament emits $q$ electrons; they begin to describe the cycloidal trajectory (for which $\alpha_0 = 1$, $\beta_0 = 0$) the collisions occurring on this trajectory will cause a new re-distribution

$$\alpha_0 S \times (\alpha_1 S + \beta_1 F) = \alpha_0\alpha_1 S^2 + \alpha_0\beta_1 SF. \qquad (16)$$

It is necessary to agree as to the meaning of the symbolic product; [12] in a game of chance $S^2$ designates two subsequent successes, $SF$ one success followed by one failure and so on. In this case there is no necessity of keeping $S$ and $F$ since the statistical weights $\alpha\beta$ already indicate the direction of scattering, namely, the statistical weight $\alpha$ indicates a $(\pi\pi)$ collision and $\beta$ indicates a $(\pi 0)$ collision. Equation (16), therefore, can be written:

$$\alpha_0 \times (\alpha_1 + \beta_1) = \alpha_0\alpha_1 + \alpha_0\beta_1$$

which means that all the electrons $\alpha_0$ transferred from the zero trajectory into the first, after the second collision undergo a new redistribution.

(1) $\alpha_0\alpha_1$ electrons go into the second trajectory ($\rho = 5$),
(2) $\alpha_0\beta_1$ electrons return back to the zero trajectory ($\rho = 1$),

the first group undergoes a new redistribution $\alpha_0\alpha_1(\alpha_2 + \beta_2) = \alpha_0\alpha_1\alpha_2 + \alpha_0\alpha_1\beta_1$ giving rise to $\alpha_0\alpha_1\alpha_2$ electrons on the third

[11] E. Borel, "Elements de la Theorie des Probabilites," Paris.
[12] E. Borel, "Theorie des Probabilites," page 15.

trajectory and $\alpha_0\alpha_1\beta_1$ electrons on the first ($\rho = 3$); the second group undergoes a redistribution $\alpha_0\beta_1(\alpha_0 + 0) = \alpha_0\beta_1\alpha_0$ which simply means that all these electrons are reflected back on the first trajectory ($\rho = 3$) since $\beta_0 = 0$.

Thus after three elastic collisions there will be

$\alpha_0\alpha_1\alpha_2$ electrons on the third trajectory ($\rho = 7$),
$\alpha_0\alpha_1\beta_1 + \alpha_0\beta_1\alpha_0$ on the first trajectory ($\rho = 3$),[13]

This step by step method can easily be applied to a gradually increasing number of collisions. The various products of statistical weights such as $\alpha_0\alpha_1\beta_1$ for example give the statistical weights of corresponding compound events. In the above example, that will mean the number of electrons which will first rise by two subsequent ($\pi\pi$) collisions from zero to the second trajectory and from there will descend by means of the third ($\pi 0$) collision to the first trajectory.

Proceeding in this manner gradually it is possible to represent graphically the probability curves attached to various levels. This is done on Fig. 1 for 28 subsequent collisions; the ordinates of the curves represent the sum of products of the statistical weights characterising the given trajectory plotted against the number of collisions; the figures attached to the curves designate electronic levels in which the distribution is represented by the probability curve. The abcissa axis does not represent any quantity proportional to the time in view of the fact that in each level there must be a factor taking into account the corresponding value of the mean free path; these factors are also different for various trajectories. This complication, however, is left out in this section[14] which is equivalent to the assumption that the collisions occur simultaneously in all levels.

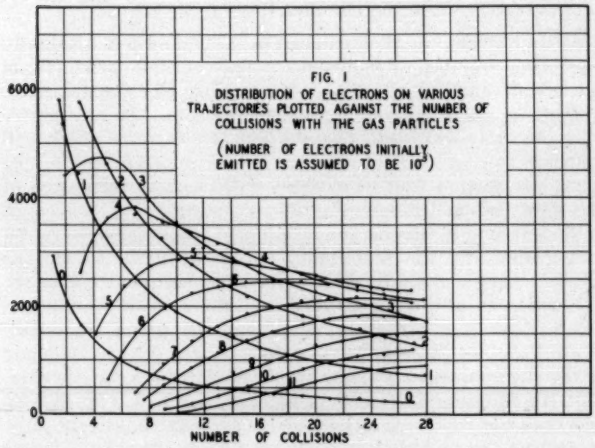
From the curves it is possible, at least qualitatively to form an idea as to how the initial layer of electrons emitted at $t = 0$ spreads itself through the gas. Near the filament the electrons drift away from the cathode comparatively swiftly since $\alpha$ weights are much greater than $\beta$ weights for these trajectories.

[13] The condition that a trajectory of an even order gives rise to two trajectories of odd orders (and vice versa) results from the assumed discreet mechanism of collisions and has no further physical significance.

[14] See the following section.

With increasing number $n$ of trajectories the difference $\alpha_n - \beta_n \to 0$ and this manifests itself statistically by a flattening out and spreading of the probability curves along the abscissa axis. The electrons have a tendency to be delayed in the gas without going promptly to the anode and the existence

FIG. 1.




of a negative space charge in the gas can be understood on this basis.

To complete the picture one should superimpose on the distribution given by curves subsequent distributions corresponding to the emission at the subsequent instants.

### SPACE CHARGE.

The preceding picture is not complete in that it considers only the probability of a given phase on trochoids at which the collisions occur. In addition to this *probability of phase* it is necessary to introduce also the *probability of collision* depending on the mean free path.

Let $x_i$ be the number of electrons lost from the $i$th trajectory per unit time by the collisions with the gas particles. With the assumed mechanism of collisions $x_i\alpha_i$ and $x_i\beta_i$ are cor-

responding numbers of electrons reflected to the nearest two trajectories on the anode and the cathode sides respectively. The probability for a group of electrons $q$ of travelling without collision through a distance $\sigma$ along the cycloidal track is $e^{-\sigma/\lambda}$ where $\lambda$ is the mean free path. During the orbital period $\tau = 1$ this trajectory will receive $q + x_1\beta_1$ electrons ($q$ electrons from the filament and $x_1\beta_1$ from the first trajectory) and $(q + x_1\beta_1)e^{-\sigma_0/\lambda}$ is the number of electrons left on the zero trajectory after the distance $\sigma_0$. But in addition to these electrons the same trajectory may be still occupied by the electrons emitted prior to the instant of the last emission.

Hence the total number of electrons present on the zero trajectory is $X_0 = \sum_{n=0}^{n=\infty} (n)(q + x_1\beta_1)e^{-\sigma_0/\lambda}$.

Using integral notation for the limit.

$$X_0 = (q + x_1\beta_1)\int_0^\infty e^{-\sigma_0/\lambda}d\tau, \tag{17}$$

where

$$\sigma = \int_0^\tau \sqrt{1 - 2\rho\cos\tau + \rho^2}\cdot d\tau$$

and

$$\sigma_0 = \int_0^\tau \sqrt{2(1 - \cos\tau)}d\tau.$$

For larger values of $\rho$, $\sigma$ can be approximated by $\rho\tau$, that is a circle substituted for the trochoid. In this case $X_0 = (q + x_1\beta_1)\,\lambda/\rho$. It is convenient to use the approximation $\sigma = \rho\tau + \epsilon(\rho)$ even for the trajectories of lower order ($\rho = 1, 3,$) by introducing a suitable function $|\epsilon(\rho)| \underset{\rho\to\infty}{\rightarrow} 0$. The calculation shows that $\epsilon(\rho)$ approaches the zero very rapidly with increasing $\rho$ and already for the third trajectory it is less than 1 per cent. of $\rho\tau$.

The average electron densities in various belts are

$$\begin{aligned}
\Delta_0 &= \frac{X_0}{2\rho_0} = (q + x_1\beta_1)\frac{\lambda}{2\rho_0^2},\\
\Delta_1 &= \frac{X_1}{2\rho_1} = (x_0\alpha_0 + x_2\beta_2)\frac{\lambda}{2\rho_1^2},\\
\Delta_n &= \frac{X_n}{2\rho_n} = (x_{n-1}\alpha_{n-1} + x_{n+1}\beta_{n+1})\frac{\lambda}{2\rho_n^2}.
\end{aligned} \tag{18}$$

In the above expressions $x_0 \ldots x_n \ldots$ are unknown. Their determination, however, is immediate in case of a steady state. Consider the $n$th trajectory: it loses $x_n\alpha_n$ electrons towards the anode and $x_n\beta_n$ towards the cathode since $\alpha_n > \beta_n$ the trajectory loses more electrons towards the anode and the difference $x_n(\alpha_n - \beta_n) = q$ is the amount of electricity transferred through the gas per unit time. Clearly

$$x_1(\alpha_1 - \beta_1) = x_2(\alpha_2 - \beta_2) = \cdots x_n(\alpha_n - \beta_n) = q.$$

Since $\alpha_n + \beta_n = 1$ for any trajectory one has

$$x_1 = \frac{q}{2\alpha_1 - 1} \cdots x_n = \frac{q}{2\alpha_n - 1}.$$

For increasing $n$ the coefficients $\alpha_n$ approach assymptotically the value 0.5, that is $x_n$ increases indefinitely. Thus the trajectories with greater generating radii lose more electrons than those of smaller radii which is physically obvious since in the former case the electrons are travelling longer paths in the gas than in the latter.

The average electron density in the $n$th belt is given by

$$\Delta_n = \left(1 + \frac{1 - \alpha_{n+1}}{2\alpha_{n+1} - 1}\right)\frac{q\lambda}{\rho_n^2} = f_n \frac{q\lambda}{\rho_n^2}; \qquad (19)$$

it is represented graphically on fig 2; it is seen that the function $\Delta_n$ is decreasing when one moves away from the cathode. The zone of the greatest electron concentration is very near to the cathode where the trajectories have rather small generating radii and therefore a rather small probability of collision with the gas particles.

The electric force in a gas is given by the first integration of the one dimensional Poisson equation in which $\eta$ is taken as independent variable instead of $\rho$.

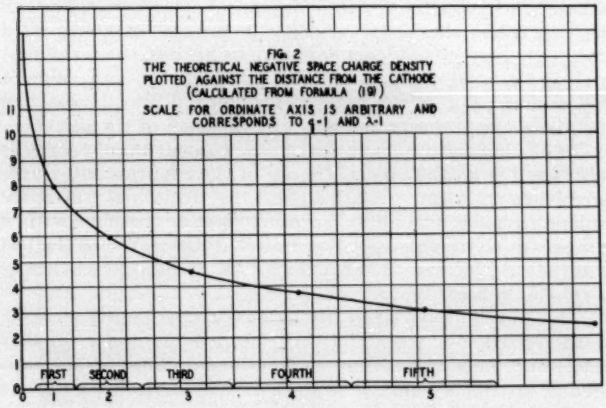
$$\frac{dV}{d\eta} = -4\pi \int_0^\eta \Delta(\eta)d\eta + \left(\frac{dV}{d\eta}\right)_0$$

and the potential

$$V = -4\pi \int_0^\eta d\eta \int_0^\eta \Delta(\eta)d\eta + \int_0^\eta \left(\frac{dV}{d\eta_0}\right)d\eta + V_0, \qquad (20)$$

where $dV/d\eta_0$ and $V_0$ are two constants of integration. One can also determine the potential difference across a gas graphically by double integration of the curve between 0 and a certain $\rho_n$. When the magnetic field is increased the size of trochoidal loops decreases so that there are more trochoidal trajectories [15] between the anode and the cathode—this is

FIG. 2.




equivalent to the increased value of the upper limit of integration which increases the value of the integral, since the integrand is an essentially positive quantity.

When the mean free path is increased, the number of trajectories between the cathode and the anode remains the same but the density in each level increases, which causes again an increase of the potential difference in the gas.

In summing up the potential difference across the gas in which the trochoidal field of electronic trajectories is established depends on two independent parameters $H$ and $\lambda$.[16]

[15] This of course relates to the discreet trajectories (without over-lapping) as is assumed in this section.

[16] See in this connection the earlier paper of the writer in Journal de Physique (April, 1928, Section 7).

With the assumed mechanism of collisions, the action of $H$ analytically corresponds to the variation of the upper limit of integration of the one dimensional Poisson's equation applied to the trochoidal field and the action of $\lambda$ to that of an independent parameter of the integrand.

### IONIZATION IN CROSSED FIELDS.

The preceding investigation can be extended to take into account the case of non elastic impacts. This corresponds to the case when the linear velocity (expressed in volts) on a trajectory becomes at least equal to, or greater than, the first critical potential of the gas. For a sufficiently high value of this orbital velocity the ionization potential may be reached and there appears a probability of ionization in the gas.[17]

In an elementary act of ionization a quantum of energy is subtracted from the kinetic energy of the electron and transferred into the atom. At this point the trochoidal trajectory undergoes a discontinuity shrinking from a trochoid with a comparatively large generating radius down to the cycloid in case the electron loses all its velocity or, at least to a trochoid of smaller generating radius.

The zone of ionization in the gas, consequently, will be characterized by trochoidal trajectories of much smaller generating radii appearing in this zone. This is also applicable to the secondary electrons released from the atoms during the process of ionization.[18]

The trochoidal field under these circumstances loses its single valued determination by equations (7). To a given level, $\eta$ does not correspond a definite $\rho$ as was the case in elastic collisions.

The model of the trochoidal field, although considerably limited in case of ionization can be still used to explain the

[17] R. Seeliger, *Ann. d. Phys.*, Bd. 59, 1919, *Zs. f. Phys.*, Bd. 2, 1920, Bd. 4, 1921, Bd. 11, 1922, Bd. 15, 1923; Seeliger, R., und Mierdel, G., *Zs. f. Phys.*, Bd. 1, 1920, Bd. 5, 1921.

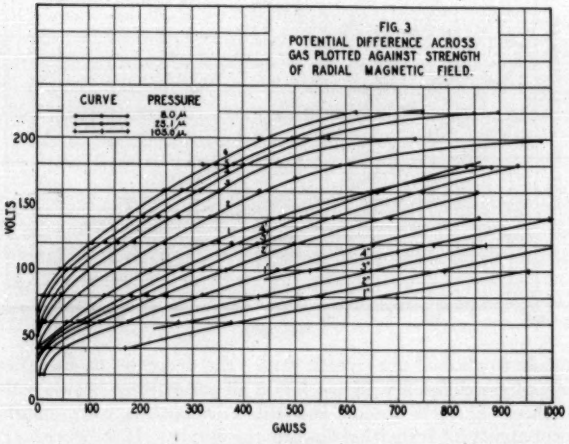
[18] The presence of positive ions modifies the space charge distribution. It is reasonable to expect, however, that this influence is not so important in the present case. In fact the positive ions owing to their small value of $e/m$ are hardly influenced by magnetic field and are not much delayed in the gas. The presence of very high potential gradients in the gas seems to corroborate also this conclusion.

existence of high potential gradients in an ionized gas placed in a transverse magnetic field.

In case of an arc placed in a radial magnetic field the existence of high potential gradients depending on two parameters $H$ and $\lambda$ was actually observed prior to the formulation of the above theory.

In case of less intense ionization in argon at comparatively low pressures[19] a similar condition exists. This is evidenced for example by the curves of Fig. 3 taken for variable $H$ and three

FIG. 3.

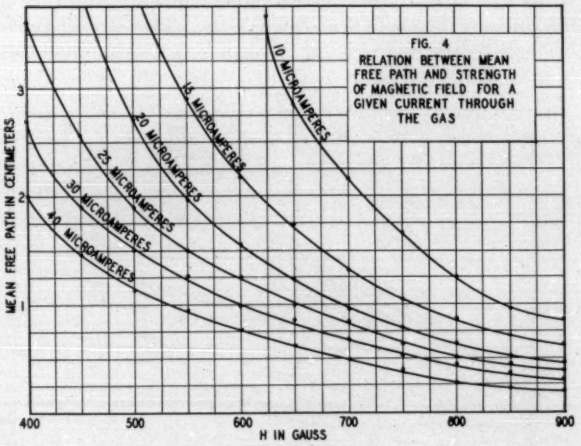



different values of $\lambda$ corresponding to the argon pressures of 8, 25 and $105\mu$ respectively. Curves 1, 2, 3, 4 etc. were taken for the currents of .1, .2, .3, etc. milliamps respectively. It is seen that for $105\mu$ the voltage across the gas increases approximately in proportion to $H$.

[19] These experiments will be described in a later publication; it is sufficient to mention here that in these experiments a circular tungsten filament was used instead of mercury in the same tube with which the arc experiments were previously made (N. Minorsky, *loc. cit.*).

The curves of Fig. 4 give the relation between $H$ and $\lambda$ for definite currents through the gas. It is seen that for increasing $H$ the mean free path decreases which is also apparent on the basis of the above theory. In fact, for the increasing $H$ the rolling radius $a = E/e/mH^2$ decreases for the

FIG. 4.




whole trochoidal field which causes the decrease, on the average of the difference $(\alpha_n - \beta_n)$ on all trajectories between the cathode and the anode and this corresponds to a smaller probability of transition toward the anode. If, however, the current through the gas must remain the same the only way to accomplish this is to act on the remaining degree of freedom viz., to increase the probability of collision by reducing the value of $\lambda$.

For a given mean free path the ionization current is the greater the stronger the field $H$ is; this increase in ionization current apparently is not due to the probability of transition on trochoidal trajectories since the magnetic field in case of elastic impacts decreases this probability of transition. One must think that this phenomenon is due rather to the increased

probability of collision which, above the ionization potential, corresponds also to the increased probability of ionization in the gas. The increase of ionization current with $H$ is much more considerable for large values of $\lambda$ than for the small ones. This seems to be also the case for the potential gradient in the gas which is much more influenced by $H$ in case $\lambda$ is large. It is very easy, for example, to raise the voltage across an arc placed in a radial field say two–three times its initial value (for $H = 0$) when the temperature is low and $\lambda$ great but, as soon as $\lambda$ is reduced, even very large changes in $H$ hardly cause any change at all on the voltage across the arc. All phenomena relative to conduction and especially to ionization in crossed fields acquire particularly interesting character for comparatively low pressures.

On the basis of the above theory it seems possible to explain this feature by a statistical process of collisions occurring in a trochoidal field between the electrons and the gas particles governed in the first approximation by two independent probabilities viz., probability of the phase depending on the magnetic field $H$ and probability of collision depending both on $H$ and $\lambda$, to which in case of ionization adds up also the probability of ionization, depending presumably on the orbital velocity of electrons.

**New Experiments on the Resistance Experienced by Fish in Moving through Water.** A. MAGNAN AND A. SAINTE-LAGUE. (*Comptes Rendus*, Nov. 12, 1929.) In order to avoid the disturbing effect of an arm or cord attached to the fish a method was devised in which all exterior connection was absent. Immediately after the death of the fish it was weighed and its buoyancy determined. Lead was placed in its mouth to make it sink head downward in water. When released it moved downward through the water without appreciable oscillation. Through a network of reference lines a moving picture of the descending fish was taken and also of a swinging pendulum to furnish time data. Twenty-two varieties of fish were thus studied. In nearly all cases the distance fallen through was quite accurately proportional to the square of the time of the fall, as is the case with a body falling freely in a vacuum. Hence, just as the falling body has a constant acceleration, so also does the descending fish. The constant acceleration indicates the action of a constant resultant force. This resultant is in quantity the excess of the weight of the fish with the added lead over the upward force due to immersion in water augmented by the force opposing the motion of the fish. Since the first two of the mentioned forces are constant and since the resultant also is constant the opposing force too is constant. This holds up to velocities of descent of 2 meters per sec. The mackerel is a fish for which the resistance is constant but for the skate the resistance increases with the velocity, as is the case with a falling sphere. G. F. S.

# THE USE OF ULTRA-VIOLET LIGHT IN THE MICROSCOPIC MEASUREMENT OF PARTICLE SIZE.*

BY

GEORGE S. HASLAM and CLYDE H. HALL,

New Jersey Zinc Company,
Palmerton, Penna.

ABSTRACT.

This article describes the new developments in mounting a pigment for photomicrography, and the use of ultra-violet light for increasing the resolution obtainable when dealing with fine particles.

Data are given that illustrate the accuracy gained in the measurement of extremely fine particles by this method.

## INTRODUCTION.

Since the limit of resolution of a microscope varies inversely as the numerical aperature of the objective and directly as the wave-length of light used, the prediction of Green [1] and others regarding the value of ultra-violet photography in the size measurement of fine particulate substances is well founded. Photomicrographs [2] have been published that show the increase in resolution obtained but no data have yet been presented to substantiate the claim for more accurate particle size measurement.

With the trend of pigment development to fine particle sizes, it becomes necessary to take advantage of the additional resolution that can be obtained with the use of shorter wave-length light. Paint and rubber pigments offer a very fertile field for an investigation of this type, as the particles lie on both sides of the limit of resolution of a high power micro-

* In presenting this method of particle size measurement, the authors acknowledge the priority of Henry Green in this field, and in making the photomicrographs for this paper have used a technique originally developed by him.

[1] Henry Green, *Chemical & Metallurgical Engineering*, Vol. XXVIII, p. 53, January, 1923.

[2] Exhibit, Royal Photographic Society, London.

scopic set-up using a 2 mm. objective, N.A. = 1.4, and essentially monochromatic light in the blue region of the spectrum ($\lambda$ = 4,480 Å).

The improvements in technique have all led to a decrease in the measured average diameter, and the authors, while unwilling to claim that no further improvements are possible, do feel that sufficient progress has been made to warrant a presentation of the technique and data at this time.

### PREPARATION OF MOUNT.

As suggested by Green, "The preparation of the mount is the really important factor involved in particle measurement. It is evident that, in order to obtain satisfactory results, the individual particles must be detached and evenly distributed; that is, all states of aggregation must be eliminated if we are interested in ultimate particle size. Second, the particles must not be broken or ground during the process of dispersion. Third, all the various sizes of particles that exist in a sample must be present in their correct proportion. Fourth, there must be a sufficient number of particles measured to give a true average. Fifth, the particles must lie in one plane and be free from motion."

The first technique described by Green [3] to fulfill these conditions is, briefly, to rub the particles out into an exceedingly thin layer with turpentine on a microscopic slide, evaporating the turpentine and mounting in glycerine or some other suitable medium. This was subsequently modified and the multiple wedge dispersion adopted.[4] This modification gave a mount in which the pigment layer varied between 0 and 25 particles in depth. With a great deal of practice and care a slide can be prepared using the latter method that will give fairly satisfactory results, and conform in general to the conditions set down above.

The technique now in use [4] is as follows: The pigment is first rubbed out in double distilled turpentine until the lumps have been disintegrated and then, when the turpentine is practically evaporated the sample is all pushed to the center

[3] Henry Green, JOUR. FRANK. INST., Nov., 1921.

[4] Henry Green, *Industrial & Engineering Chemistry*, Vol. XVI, No. 7, p. 667, July, 1924.

of the slide and the remainder of the turpentine is driven off with heat. It is extremely important that all the pigment be gathered together in the center of the slide, for in the rubbing out operation the coarse particles are carried to the edges of the slide and would be lost in the final mount.

After the turpentine has been evaporated, the slide is allowed to cool, and all the excess pigment is wiped off with lens paper, leaving a square about .5 cm. in the center of the slide. The slide is then returned to the hot plate and a small piece of dammar gum (about 5 mg.) is placed on the square of pigment and allowed to melt. The cover glass is placed over the dammar and gently moved about. The pigment is picked up and dispersed in the melted gum.

When dispersion is complete the slide is removed from the hot plate and, by holding the cover glass in place with a pair of forceps and rubbing with a blunt stick while the gum is hardening, it is possible to form a wedge of pigment varying in thickness from no particles to a layer fifteen or twenty particles deep.

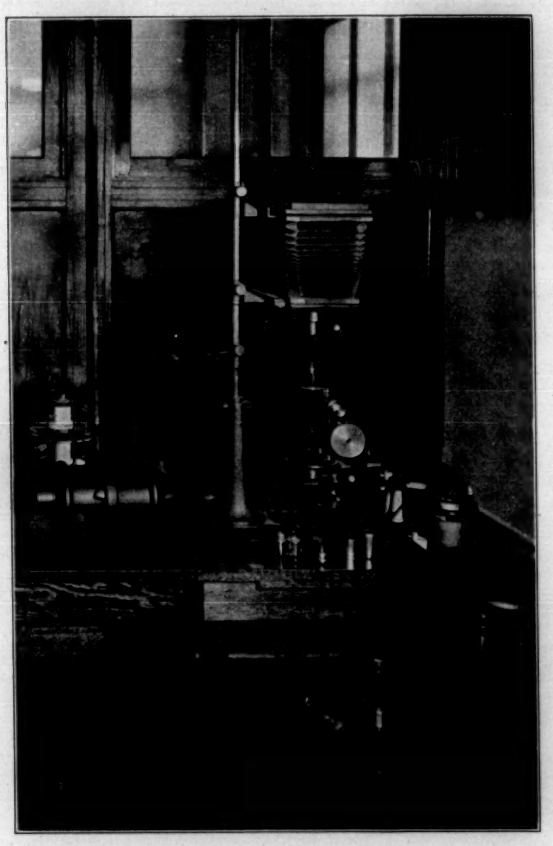
This mount has several advantages in that it is far simpler to prepare successfully, better dispersion of the pigment is assured, a more permanent mount is prepared and, owing to the rigidity of the dammar, it lends itself to easier manipulation of the high power objectives on the microscope.

In preparing a mount for ultra-violet photomicrography, the same procedure is followed, using a quartz slide and cover slip. Care must be taken to place the cover slip over the dammar before it melts, as melted dammar oxidizes rapidly in air and in the oxidized condition becomes quite opaque to ultra-violet light.

### APPARATUS AND PHOTOGRAPHY.

The equipment used is the Zeiss ultra-violet microscope, as shown in Fig. 1, which is adaptable for both visible and ultra-violet examination. For visible light the optical system is glass throughout, and for ultra-violet light is quartz which has been corrected for light of $\lambda = 2{,}750$ Å. The source of illumination is a spark between two metal electrodes; magnesium for visible light and cadmium for ultra-violet light. This light is passed through dispersing prisms, and

FIG. 1.



The ultra-violet microscope.

only that light which is being used is allowed to enter the microscope. For visual observation, $\lambda = 4{,}480$ Å. and for ultra-violet light photography $\lambda = 2{,}750$ Å. have been selected as giving the most satisfactory results.

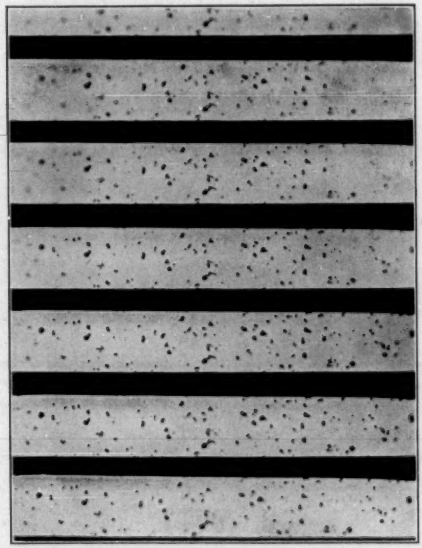
When the pigment has been mounted as described on page 778, in dammar on a glass microscopic slide, it is photographed using a 2 mm. objective with 1.4 N.A. and a 6 X ocular. This is done, using the blue light illumination and focusing the pigment particles on a ground glass plate at the plate holder.

The photography with ultra-violet light is not so simple. The sample is prepared on a quartz microscopic slide as described above and then placed on the microscope, which is first set up with a glass optical system. The blue light of the magnesium electrodes is used as a source of illumination to center the optical system and for the selection of a representative field. Then the glass objective and ocular are replaced by the quartz objective and ocular, and the focus again determined. Then the quartz condenser is substituted for the glass and centered, still using visible light. The glass dispersing prisms are removed and the quartz prisms put in their place, and the prism table is turned about its vertical axis until the series of ultra-violet lines at approximately 2,800 Å., given off by the magnesium spark, are visible in the total reflecting prism at the base of the microscope. A piece of uranium glass which fluoresces in ultra-violet light is used to detect this light. The microscope is then racked down 2.3 revolutions of the fine adjustment from the blue light setting, a distance known to bring the sample in focus for the ultra-violet light. The condenser is then focused and the optical system is properly adjusted, which can be done only after considerable practice and experimentation. All the adjustments are made by means of the searcher eyepiece, which consists of a field lens of quartz above which is mounted a piece of uranium glass, the fluorescence of which is magnified by a glass lens. All this has been done with the ultra-violet band of the magnesium spark because of its greater intensity, rather than with the single line of the cadmium spark.

The magnesium electrodes are then exchanged for the cadmium which gives off an ultra-violet line at 2,750 Å. The

condenser is more carefully focused and the illumination properly adjusted. Trial exposures at different settings of the fine adjustment are made by means of a sliding plate holder and the proper focus selected for a final picture. A print showing the trial exposures is shown in Fig. 2.

FIG. 2.




Showing strip exposure used for zinc oxide No. 2 in locating correct focus with ultra-violet light. "A" is in proper focus.

### SCOPE OF EXPERIMENT.

Three different zinc oxides were selected for this work and are designated as Zinc Oxide No. 1, Zinc Oxide No. 2 and Zinc Oxide No. 3.

Zinc Oxide No. 1 is a sample of very coarse, but quite uniform pigment prepared by calcining zinc oxide at 900° C.

All the particles are known to be quite coarse because in a water suspension the particles scatter little if any light. The measurement of the particle size of this pigment has presented no difficulties to the visible light technique and was included in this investigation for comparison purposes.

Zinc Oxide No. 2 is an American process oxide which is known to contain some fines (below .2 microns). The presence of fine particles is readily detected by passing a beam of light through a suspension of the pigment and noting the intensity of the Tyndall beam. This sample has, however. a range of particles from below .2 microns to 1.5 or 2.0 microns,

Zinc Oxide No. 3 is an extremely fine pigment and is quite uniform. It is as a test for the accurate determination of the size of this pigment that the present technique is being developed. A large percentage of the particles lie below the resolving power of the high power microscopic set-up, and photographic efforts have all resulted in obtaining nothing more than diffraction patterns, rather than true images of the pigment particles.

A comparison has been made by preparing mounts of these three pigments by the two methods mentioned on the preceding page, namely, the turpentine dispersion and the dammar dispersion. The turpentine dispersion was photographed, using essentially the mono-chromatic blue light of a magnesium spark ($\lambda$ = 4,480 Å.) as a source of illumination. The dammar mount was photographed with this same source of illumination and then with ultra-violet light (monochromatic light, $\lambda$ = 2,750 Å.) using a cadmium spark as a source of illumination. It was, of course, impossible to photograph the same field for in the case of the blue light it was essential that a glass microscopic slide be used, in order not to interrupt the homogeneity of the optical system. And, obviously, for the ultra-violet work a glass slide could not be used.

The photomicrography was done at a magnification of 1,285 and then enlarged to 3,860 ×. Prints were prepared and the particle size measured, taking the horizontal diameter of the particle as its average diameter.[6]

[6] Martin Geoffrey, *Trans. Cer. Soc.*, 23, p. 61, 1923.

DISCUSSION OF DATA.

The following table shows the results obtained from the measurement of the particle size of the pigments as shown in the preceding photomicrographs and curves:

TABLE OF RESULTS.

| | Illumination. | Dispersion. | Measured Av. Particle[1] Size in Microns. | | Specific Surface Sq. In. Per Gm. |
|---|---|---|---|---|---|
| | | | $d_1$. | $d_3$. | |
| Zinc Oxide No. 1 | Blue light | Turpentine | .442 | .603 | 1.74 |
| | Blue light | Dammar | .317 | .523 | 2.02 |
| | Ultra-violet light | Dammar | .300 | .542 | 1.93 |
| Zinc Oxide No. 2 | Blue light | Turpentine | .285 | .478 | 2.20 |
| | Blue light | Dammar | .228 | .477 | 2.20 |
| | Ultra-violet light | Dammar | .189 | .393 | 2.68 |
| Zinc Oxide No. 3 | Blue light | Turpentine | .188 | .267 | 3.94 |
| | Blue light | Dammar | .179 | .256 | 4.10 |
| | Ultra-violet light | Dammar | .164 | .246 | 4.27 |

In considering Zinc Oxide No. 1 (See Fig. 3, Curve I) it will be noted that while there is a fairly large discrepancy between the turpentine and dammar mounts which is undoubtedly a question of dispersion, there is practically no difference between the blue light and the ultra-violet light. measurements, indicating that the additional resolution gained has not been sufficiently great to warrant the use of the refined method.

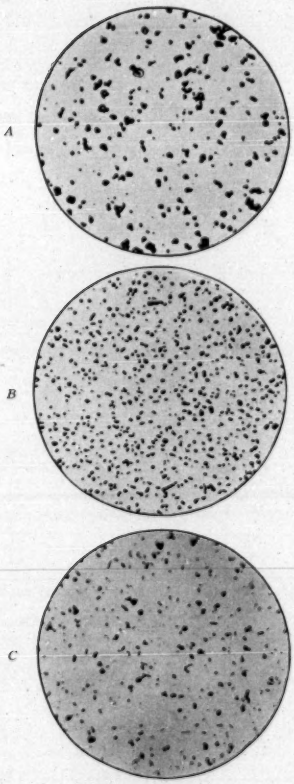
Zinc Oxide No. 2 (See Fig. 4, Curve II) containing as it does a certain percentage of fines, shows a fairly large difference between the measured average diameter with blue light and with ultra-violet light. This difference can be attributed to the fact that with the shorter wave-length, and, therefore, additional resolution, it has been possible to show more of the fines, and to have particles which appear in blue light as diffraction patterns, which must be larger than the true image, appear in the ultra-violet light at the true size.

Zinc Oxide No. 3 (See Fig. 5, Curve III), a colloidal zinc oxide, has, with ultra-violet light, given results that are

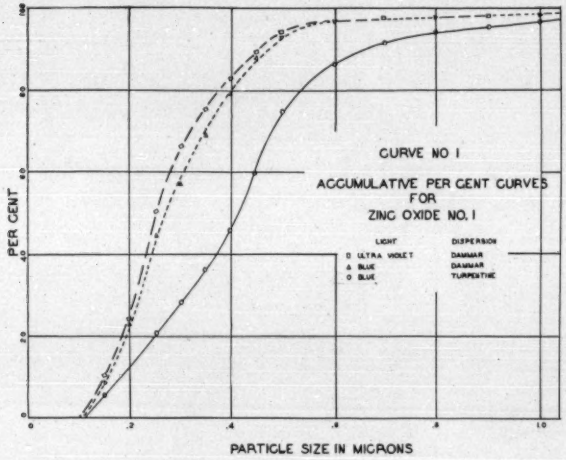
[1] Perrott and Kinney, *Journal American Ceramic Society*, 1922, 17, 327.

FIG. 3.

11A




Magnification = 1285 X.
Zinc Oxide No. 1


CURVE NO 1
ACCUMULATIVE PER CENT CURVES
FOR
ZINC OXIDE NO. 1
LIGHT
DISPERSION
□ ULTRA VIOLET
DAMMAR
△ BLUE
DAMMAR
○ BLUE
TURPENTINE
PER CENT
PARTICLE SIZE IN MICRONS

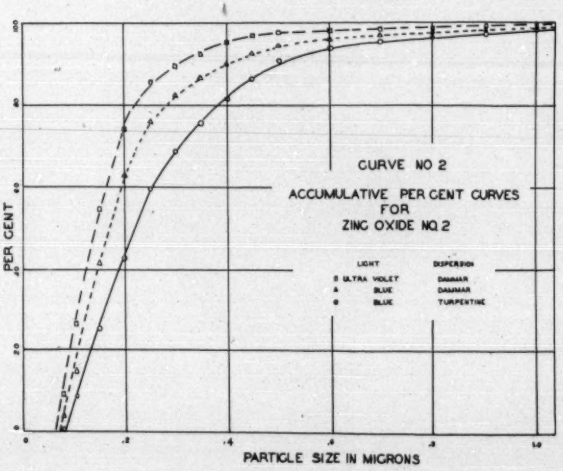
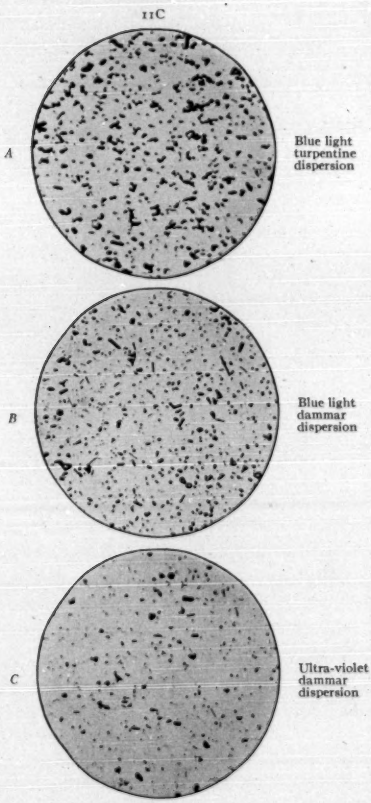
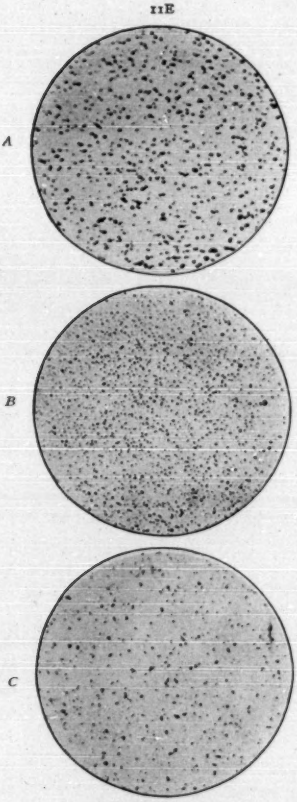

CURVE NO 2
ACCUMULATIVE PER CENT CURVES
FOR
ZINC OXIDE NO 2
LIGHT
DISPERSION
□ ULTRA VIOLET
DAMMAR
△ BLUE
DAMMAR
○ BLUE
TURPENTINE
PER CENT
PARTICLE SIZE IN MICRONS

FIG. 4.



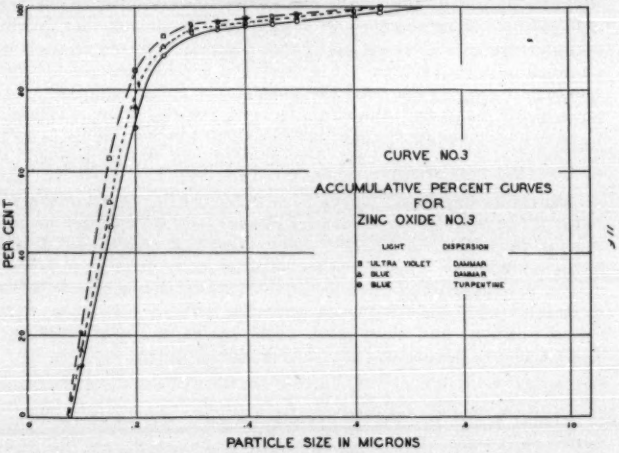

Magnification = 1285 ×.
Zinc Oxide No. 1.

FIG. 5.




Magnification = 1285 X.
Zinc Oxide No. 3.

far more satisfactory than those that have been obtained with blue light, but it must be admitted that the ultimate has not been obtained. The photograph still shows the presence of diffraction circles, indicating the presence of fines that are still below the resolving power of the ultra-violet microscope.




The authors do not feel that the quartz objectives have reached the same degree of perfection that the glass objectives have, and that with the development of a high numerical aperture quartz objective, even better resolutions may be obtained.

SUMMARY.

Photomicrographs have been obtained using monochromatic ultra-violet light that are far more satisfactory than those taken with blue light.

Particle size measurements have been made that indicate that measurements made by ordinary methods on pigments containing colloidal particles give values for the average diameters that are too large.

Ultra-violet light is an aid in the measurement of the particle size of pigments of colloidal dimensions, but has no advantage when dealing with the coarser pigments.

**Evaporated Milk.** According to FRANK E. RICE (*Am. J. Public Health*, 1929, *19*, 777–781) the annual consumption of concentrated and preserved milks in the United States has increased from less than one billion pounds in 1908 to four and one-third billion pounds. The chief forms are evaporated milk, condensed milk (containing added sucrose), dry milk, and malted milk. The annual consumption of evaporated milk per annum in the United States is one and one-quarter billion pounds. It is used as an article of diet, in the manufacture of ice cream and candy, and in the preparation of modified milk for infants. It is described as safe, dependable and economic. Its calcium and phosphorus are fully assimilable; its fat is highly emulsified; the vitamins, with the exception of vitamin C, have undergone no change; and the curd, which it yields, is soft.

J. S. H.

**Starchlike Crystals.** LUCIA MCCULLOUGH (*J. Agric. Research*, 1929, *39*, 495–501) describes starchlike crystals which form in cultures of *Bacterium marginatum* when starch is a constituent of the nutrient medium. They are spherocrystals, occur in great numbers, may have a diameter as great as 140 microns, consist of needlelike or threadlike parts radiating from a center, yield a blue color with iodine solution, and show dark cross marks in polarized light. Their chemical nature has not yet been determined.

J. S. H.

**Atomic Weights.** The Thirty-sixth Annual Report of the Committee on Atomic Weights (*J. Am. Chem. Soc.*, 1930, *52*, 857–862) is written by GREGORY PAUL BAXTER and contains the table of atomic weights for 1930. The only change in the table is a new and lower value for the atomic weight of arsenic 74.93.

J. S. H.

# STRESSES IN HEAVY HELICAL SPRINGS.

BY

ELMER LATSHAW.

An important application of heavy helical springs is in the spring suspension of railroad rolling stock. In such service there is static loading due to the equipment and lading weight, also dynamic loading resulting from impact when the wheels roll over track irregularities and special track work.

The stress in the spring, when subjected to maximum static and dynamic loading, must be under the elastic limit of the material. How close to the elastic limit it is permissible to allow the maximum stress to approach, depends on spring life expected, also how often the stress reaches this maximum value.

In a given spring, stress is directly proportional to load and deflection, thus the energy stored will vary directly as the stress squared. Therefore to attain high spring efficiency it becomes very necessary that the stress be the highest possible for a reasonable spring life. Working to high stresses requires, in addition to materials of uniform properties, an accurate knowledge of the stress present.

There is evidence of opportunity for an improvement in helical spring stress formulæ such as are in common use. A spring tested statically will take a permanent set when stressed considerably below the elastic limit of the material according to present formula.

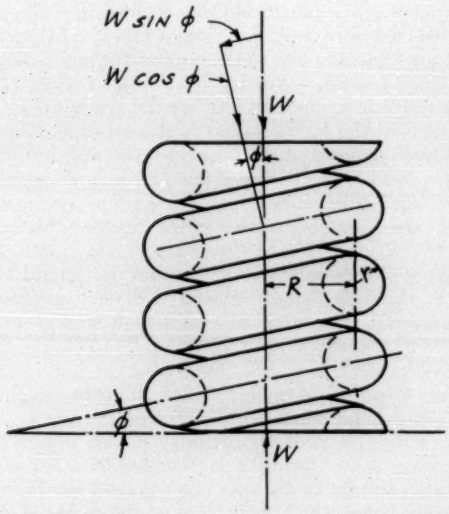
If springs of all proportions would take a permanent set at the same stress the common formula might be used with a suitable constant so as to give actual stress values, but such is not the case. Experience has shown that springs of low index have much shorter life than those of higher index when supposedly stressed equally and in similar service.

The common stress formula gives only the supposed torsional stress which is shear, or direct stress inclined 45° to

the shear stress. It does not include the several secondary stresses or any modification for helix angle or corrections in stress distribution due to bar curvature. A more exact stress formula will now be derived where in all these points will be considered.

Heavy helical springs, in the most used form, are made of round bar and wound open coiled. (See Fig. 1.) The loading

FIG. 1.




is compressive and axially applied thereby decreasing the helix angle with increased load. Adjacent turns of the helix are separated in an unstressed state by an amount such that when loaded to maximum capacity the coils close, thus preventing accidental additional stress. In the subsequent work only springs made of round section bar will be considered.

SYMBOLS.

$R$ = Pitch radius of helix at bar center.
$r$ = Bar radius.
$\frac{R}{r} = i$ = Spring index.
$A = \pi r^2$ = Area of bar.
$W$ = Axial loading (compressive).
$\phi$ = Pitch angle of helix.
$p$ = Direct unit stress (Tensile or compressive).
$q$ = Shear unit stress.
$f$ = Combined unit stress.
$E$ = Direct modulus of elasticity.
$N$ = Shear modulus of elasticity.
$I$ = Moment of inertia (bending).
$J$ = Moment of inertia (polar).

Other symbols will be defined as used.

Resolving the axial loading $W$ on a normal section of the bar which is at right angles to the bar center and inclined by an angle $\phi$ to the base line gives the relations (see Fig. 1):

$$\text{Shear} = F = W \cos \phi, \tag{1}$$

$$\text{Compression} = C = W \sin \phi. \tag{2}$$

If the effects of bar curvature on stress are neglected the following can be written:

$$\text{Torque} = T = WR \cos \phi, \tag{3}$$

$$\text{Bending} = M = WR \sin \phi. \tag{4}$$

Dividing (3) by the polar section modulus of a round bar $Ar/2$ and letting ($\phi = 0$) gives the common stress formula:

$$p = q = \frac{2Wi}{A}, \tag{5}$$

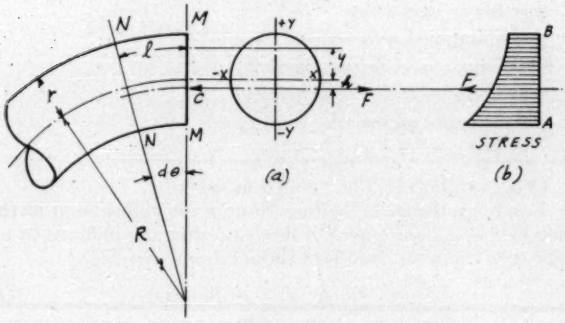
where $p$ is unit direct stress on planes inclined 45° to planes of $q$.

Formula (5) will be recognized as the stress formula which is quite generally used. The approximations made to derive

(5) indicate its limitations for accurately giving even the stress resulting from torsion. A complete study of the stresses should include stresses from all the component forces and moments given by formulæ (1) to (4) with proper modifications in stress distribution due to bar curvature.

Bar curvature induces modifications in unit stress variation, from that in a straight bar, due principally to the variable distance between adjacent normal bar sections at different points over the area. In Fig. 2 $a$ denoting the distance be-

FIG. 2.




Forces F & C.

tween sections $MM$ and $NN$ by $l$ at a point distance $y$ from the bar center gives:

$$l = d\theta(R + y)$$

and for $y = 0$ let $l' = Rd\theta$ hence

$$l = l'\left(\frac{R + y}{R}\right). \tag{6}$$

*Shear.*—The shear component $F$ acts tangent to a normal section thereby displacing section $MM$ relative to $NN$ an amount denoted by $s$ which is due to shear strain action. The displacement $s$ is in the direction of $F$ and parallel to the normal section and for pure shear is evidently equal over the entire area.

The shear strain at any point is $s/l$ therefore the unit shear stress is $q = Ns/l$. Substituting the value of $l$ as given by (6) gives:

$$q = \frac{Ns}{l} = \frac{Ns}{l'}\left(\frac{R}{R+y}\right) = q'\left(\frac{R}{R+y}\right), \tag{7}$$

Where $q'$ is the unit shear stress at the bar center.

Summing up the shear resistance of the entire bar area gives:

$$F = \Sigma q da = q'\Sigma\left(\frac{R}{R+y}\right)da. \tag{8}$$

The summation $\Sigma\left(\frac{R}{R+y}\right)da$ is derived as follows:

$$\Sigma\frac{R}{R+y}da = 2R\int_{-r}^{+r}\frac{\sqrt{r^2-y^2}}{R+y}dy = 2\pi R(R-\sqrt{R^2-r^2})$$
$$= 2\pi r^2 i(i-\sqrt{i^2-1}) = A2i(i-\sqrt{i^2-1}).$$

Letting $k = 2i(i-\sqrt{i^2-1})$, then

$$\Sigma\frac{R}{R+y}da = Ak. \tag{9}$$

Substituting the value of $q'$ from (7) and $\Sigma\frac{R}{R+y}da$ from (9) in (8) gives:

$$F = q\left(\frac{R+y}{R}\right)Ak, \tag{10}$$

$$q = \left(\frac{F}{A}\right)\frac{1}{k}\left(\frac{R}{R+y}\right). \tag{10a}$$

Formula (10$a$) gives the variation of unit shear stress over the section which is shown graphically by figure 2$b$.

The maximum stress at $A$ and minimum at $B$ is given below:

$$q_A = \frac{W\cos\phi}{Ak}\left(\frac{i}{i-1}\right), \tag{10c}$$

$$q_B = \frac{W\cos\phi}{Ak}\left(\frac{i}{i+1}\right). \tag{10d}$$

The unit shear stress being variable over the area and greatest towards the helix axis causes the total shear resistance $F$ to act at a position denoted by $h$ from the bar center. The total moment of shear resistance about the center of gravity of $F$ is zero therefore $h$ can be found by writing:

$$\Sigma q(y - h)da = q'\Sigma \frac{R(y - h)}{R + y} da$$

$$= q'\Sigma \left( R - \frac{R^2}{R + y} - \frac{Rh}{R + y} \right) da = 0.$$

Using relation given by (9) yields:

$$0 = R - Rk - hk$$

and transposing

$$h = \left( \frac{R}{k} - R \right). \tag{11}$$

The distance from the point of action of $F$ to helix axis is

$$(R + h) = \frac{R}{k}. \tag{11a}$$

Other values of $h$ useful later are

$$h = -\frac{R}{k}(k - 1). \tag{11b}$$

Since $k = 2i(i - \sqrt{i^2 - 1})$ and $k^2 = 4i^2(k - 1)$,

$$h = -\frac{Rk}{4i^2}. \tag{11c}$$

*Compression.*—The compression component $C$ acts perpendicular to a normal section setting up direct stresses between sections $MM$ and $NN$ which gives uniform straining action over the section for pure compression which will be denoted by $e$.

The direct stress (compression) at any point is:

$$p = \frac{Ee}{l} = \frac{Ee}{l'} \left( \frac{R}{R + y} \right) = p' \left( \frac{R}{R + y} \right). \tag{12}$$

Summing the resistance gives:

$$C = \Sigma pda = p'\Sigma\left(\frac{R}{R+y}\right)da = p'Ak = pAk\left(\frac{R+y}{R}\right) \quad (13)$$

and

$$p = \left(\frac{C}{A}\right)\frac{1}{k}\left(\frac{R}{R+y}\right). \quad (14)$$

Formula (14) gives the variation of unit direct stress over the section which is shown graphically by figure 2*b*. The maximum stress at $A$ and minimum at $B$ is given below, using the negative sign to denote compression thus:

$$p_A = -\frac{W\sin\phi}{Ak}\left(\frac{i}{i-1}\right), \quad (14a)$$

$$p_B = -\frac{W\sin\phi}{Ak}\left(\frac{i}{i+1}\right). \quad (14b)$$

The variable part of (14) being the same as in (10*a*) will likewise give the position at which $C$ acts as distance $h$ from the bar center.

Thus bar curvature shifts the point at which $W$ is resolved into $F$ and $C$ off the bar center a distance $h$ towards the helix axis, also the unit stresses from $F$ and $C$ are caused to vary otherwise than uniformly over the area, being greatest at the diametrical extremity towards the helix axis. The modification in stress distribution and value of $h$ evidently varies with the spring index, $i$, being greatest for small values of $i$.

*Bending.*—The bending moment $M$ sets up direct stresses over the bar area which are tensile and compressive on opposite sides of the neutral axis. Bar curvature affects the stress at a point $y$ between sections $MM$ and $NN$ from a strain $e$ as given by:

$$p' = \frac{Ee}{l} = \frac{Ee}{l'}\left(\frac{R}{R+y}\right) = p''\left(\frac{R}{R+y}\right), \quad (15)$$

wherein $e$ is assumed constant.

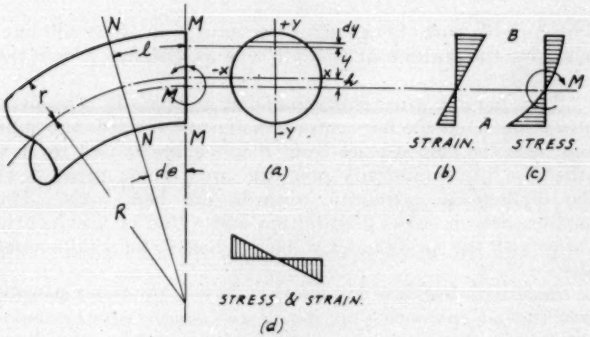
In a straight bar stress is proportional to $e$ only as the distance between normal sections is uniform over the area hence the center of gravity or center about which bending takes place equals $p\Sigma yda/p\Sigma da$ where $p$ cancels out. To find the center of gravity of a curved bar the moment will be taken about the helix axis, thus:

$$\frac{\Sigma p'(R+y)da}{\Sigma p'da} = \frac{p''\Sigma Rda}{p''\Sigma\left(\frac{R}{R+y}\right)da} = \frac{RA}{kA} = \frac{R}{k}. \quad (16)$$

Note that $R/k = (R+h)$ by (11$a$) and that $h$ is negative in sign. Thus bending takes place distance $h$ from bar center towards helix axis or at the point where $W$ is resolved into $F$ and $C$.

Simple bending sets up strains proportional to the distance from the neutral axis or center of gravity (see figure 3$b$ and $d$) since any normal section is assumed to be plane before and after stressing, as in straight bars. Letting the strain at $(y-h)$ equal $e$ and at $(y=0)$ be $e'$ thus:

FIG. 3.




Bending moment M.

$$e = e'\left(\frac{y-h}{h}\right)$$

therefore the direct stress at $(y - h)$ will be

$$p = \frac{Ee}{l} = \frac{Ee'}{hl'}\left(\frac{y-h}{R+y}\right)R = \frac{p''}{h}\left(\frac{y-h}{R+y}\right)R. \qquad (17)$$

Summing the moment of resistance gives:

$$M = \Sigma p(y-h)da = \frac{p''}{h}\Sigma\frac{R(y-h)^2}{R+y}da = \frac{p''I_z}{h}. \qquad (18)$$

Where $I_z$ is the moment of inertia for a curved bar.

Substituting the value of p'' from (17)

$$M = \frac{p}{R}\left(\frac{R+y}{y-h}\right)I_z, \qquad (19)$$

$$p = \frac{MR}{I_z}\left(\frac{y-h}{R+y}\right). \qquad (20)$$

The graph of bending stress from (20) is shown in figure 3*c*. The algebraic value of $I_z$ can be found by the summation:

$$I_z = \Sigma(y^2 - 2yh + h^2)\left(\frac{R}{R+y}\right)da \qquad (21)$$

$$= \Sigma\left(yR - R^2 + \frac{R^3}{R+y} - 2hR + \frac{2hR^2}{R+y} + \frac{h^2R}{R+y}\right)da.$$

Applying equations (9) and (11*a*)

$$I_z = -AhR. \qquad (21a)$$

Using the value of $h$ from (11*c*)

$$I_z = \frac{Ar^2k}{4}. \qquad (21b)$$

Using the value of $I_z$ given by (21*b*) in stress formula (20)

$$p = \frac{4MR}{Ar^2k}\left(\frac{y-h}{R+y}\right). \qquad (22)$$

The bending moment $M$ tends to unwind the spring helix or increase the radius $R$ thereby setting up tensile stress towards the helix axis and compression outside the neutral axis. Maximum tensile stress occurs at point $A$ where $(y = -r)$ and maximum compression stress occurs at $B$ where

$(y = +r)$. Therefore to denote tension by positive signs the moment $M$ must be considered negative and the maximum values therefore are:

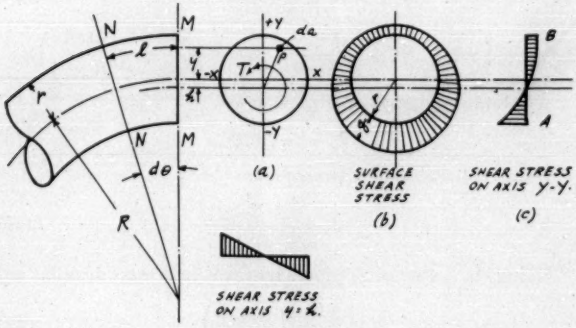
$$\text{Tension} \qquad p_A = \frac{W \sin \phi}{Ak}\left(\frac{i}{i-1}\right)\left(\frac{4i-k}{k}\right); \qquad (22a)$$

$$\text{Compression} \quad p_B = -\frac{W \sin \phi}{Ak}\left(\frac{i}{i+1}\right)\left(\frac{4i+k}{k}\right). \qquad (22b)$$

In the above $WR \sin \phi/k$ is substituted for $M$ where $R/k$ is the moment arm of the force $W \sin \phi$ and is the distance from the helix axis to the center of bending. The value of $h$ as of (11$c$) is used to reduce these formulæ to the form given.

*Torsion.*—Torsional strain action will take place about a point at the intersection of the two neutral axes in bending. For bending about the $\chi$ axis the neutral axis is at $y = h$ as shown previously. For bending about the $\gamma$ axis it is evidently at $x = 0$ since an element $2xdy$ is of uniform length

FIG. 4.




(d)

Torque T.

between normal sections (see figure 3$a$), the stress and strain graph being shown by figure 3$d$. The shear strain action will vary in intensity directly as the radius $\rho$ from the neutral center $(y = h)$ and $(x = 0)$. Denote the strain at $\rho$ by

$s/l$ and at $(\rho = h)$ by $s'/l$ (see figure 4$a$) then

$$q = \frac{sN}{l} = \frac{s'\rho N}{lh} = \frac{s'\rho N}{l'h}\left(\frac{R}{R+y}\right). \tag{23}$$

Let $q' = s'N/l'$ which is the shear stress at the bar center, thus

$$q = q'\frac{\rho}{h}\left(\frac{R}{R+y}\right). \tag{24}$$

Summing the torsional resistance gives:

$$T = \Sigma q\rho da = \frac{q'}{h}\Sigma\rho^2\left(\frac{R}{R+y}\right)da = \frac{q'}{h}J, \tag{25}$$

Where $J$ is the polar moment of inertia for a curved bar.

Using the value of $q'$ from (24) in (25) gives:

$$T = \frac{qJ}{\rho}\left(\frac{R+y}{R}\right), \tag{25a}$$

$$q = \frac{T\rho}{J}\left(\frac{R}{R+y}\right). \tag{25b}$$

The value of $J$ for a round curved bar is found as follows:

$$\begin{aligned} J &= \Sigma\rho^2\frac{R}{R+y}da = \Sigma[(y-h)^2 + x^2]\frac{R}{R+y}da \\ &= I_x + \Sigma x^2\left(\frac{R}{R+y}\right)da \\ &= I_x + \int_{-r}^{+r}\int_{-\sqrt{r^2-y^2}}^{\sqrt{r^2-y^2}} x^2\left(\frac{R}{R+y}\right)dydx \\ &= I_x + \frac{1}{3}\Sigma(r^2 - y^2)\left(\frac{R}{R+y}\right)da \\ &= I_x + \frac{1}{3}\Sigma\left(\frac{r^2R}{R+y} - yR + R^2 - \frac{R^3}{R+y}\right)da, \end{aligned}$$

applying equations (9), (11$b$) and (11$c$) gives:

$$J = \frac{Ar^2k}{4} + \frac{Ar^2k}{12}(4-k)$$

$$J = \frac{Ar^2k(7-k)}{12}.$$

Using this value of $J$ in formula (25b)

$$q = \frac{12T}{Ar^2k}\left(\frac{R}{R+y}\right)\frac{\rho}{(7-k)}. \tag{26}$$

The variable radius $\rho$ can be expressed in terms of $x$ and $y$

$$\rho^2 = y^2 - 2yh + h^2 + x^2.$$

Letting $(x = 0)$ gives $\rho = (y - h)$ which when used in (26) gives the stress variation along the diameter $\gamma - \gamma$ shown graphically by figure 4*c*. Letting $(y = h)$ gives $\rho = x$ which used in (26) gives the stress along the line distance $(y = h)$ from bar center shown by figure 4*d*. The stress at the bar surface is shown by figure 4*b* and found by using $r^2 = x^2 + y^2$ thus $\rho^2 = r^2 - 2yh + h^2$.

Thus from torsion the shear stress on diameter $\gamma - \gamma$ is:

$$q = \frac{12T}{Ar^2k}\left(\frac{R}{R+y}\right)\left(\frac{y-h}{7-k}\right). \tag{27}$$

The sign of shear stress from $F$ is assumed positive in sign and to make the shear from $T$ at a point $A$ where $y = -r$ also positive the sign of $T$ must be assumed negative as it is obvious that shear stresses from $F$ and $T$ add at point $A$ and subtract at point $B$. Therefore the shear from $T$ at $A$ and $B$ will be:

$$q_A = \frac{W\cos\phi}{Ak}\left(\frac{i}{i-1}\right)\left(\frac{4i-k}{7-k}\right)\frac{3}{k}, \tag{27a}$$

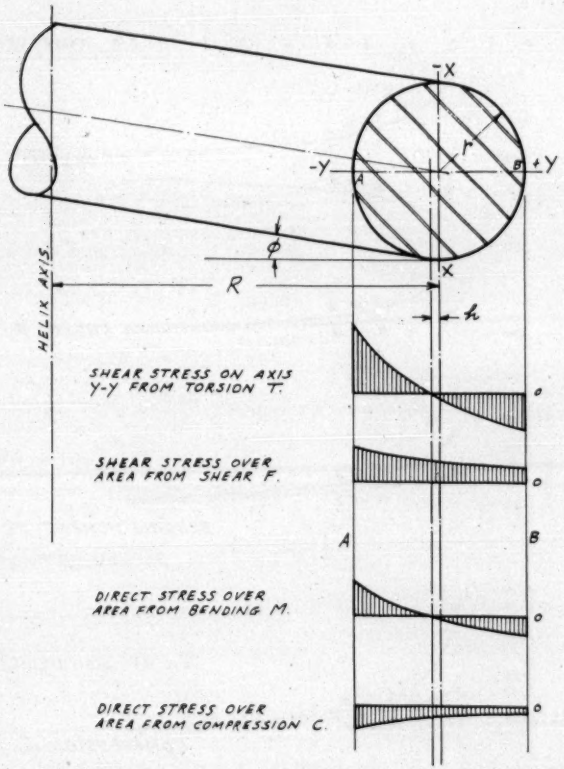
$$q_B = -\frac{W\cos\phi}{Ak}\left(\frac{i}{i+1}\right)\left(\frac{4i+k}{7-k}\right)\frac{3}{k}. \tag{27b}$$

In above formulæ $T = WR\cos\phi/k$ is used where $R/k$ is the distance from the helix-axis to the center of torsion and the value of $h$ as given by (11*c*) is used.

Figure 5 shows graphically the variation of the four component stresses across the diameter $\gamma - \gamma$. The maximum combined stress will occur at point $A$ and the minimum at point $B$. The pitch angle $\phi$ is a variable since it changes with increased applied load and is obviously a minimum when the spring is compressed solid. It is customary in spring design that the pitch be such that adjacent coils come in

contact or the spring becomes solid when loaded to maximum capacity. Thus the relation $\tan \phi = 1/\pi i$ can be written

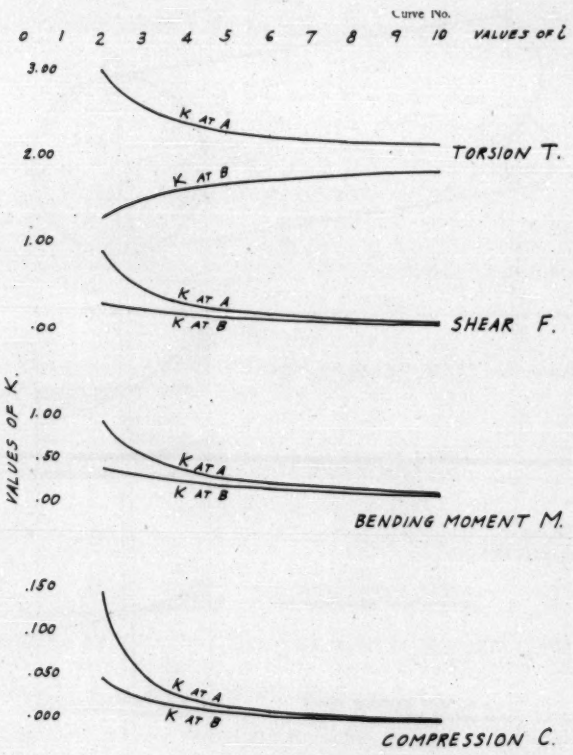
FIG. 5.




when maximum loading and stress occur. Therefore all the foregoing stress formulæ for points $A$ and $B$ can be reduced to the form $wiK/A$ where the value of $K$ is dependent entirely

on the spring index $i$. Figure 6 shows the graph of $K$ for the four component stresses disregarding signs at points $A$ and $B$

FIG. 6.




Component stresses.

plotted against values of $i$. To clearly show the formulæ used to plot graphs in figure 6 these will be rewritten.

FOR POINT A.

From Shear $F$,

$$q = \frac{WiK}{A}, \quad \text{where} \quad K = \frac{\cos \phi}{k(i-1)}, \tag{10c}$$

From Torque $T$.

$$q = \frac{WiK}{A}, \quad \text{where} \quad K = \frac{3 \cos \phi (4i-k)}{k^2(i-1)(7-k)}, \tag{27a}$$

From Bending $M$,

$$p = \frac{WiK}{A}, \quad \text{where} \quad K = \frac{\sin \phi (4i-k)}{k^2(i-1)}, \tag{22a}$$

From Compression $C$,

$$p = -\frac{WiK}{A}, \quad \text{where} \quad K = \frac{\sin \phi}{k(i-1)}. \tag{14a}$$

FOR POINT B.

From Shear $F$,

$$q = \frac{WiK}{A}, \quad \text{where} \quad K = \frac{\cos \phi}{k(i+1)}, \tag{10d}$$

From Torque $T$,

$$q = -\frac{WiK}{A}, \quad \text{where} \quad K = \frac{3 \cos \phi (4i+k)}{k^2(i+1)(7-k)}, \tag{27b}$$

From Bending $M$,

$$p = -\frac{WiK}{A}, \quad \text{where} \quad K = \frac{\sin \phi (4i+k)}{k^2(i+1)} \tag{22b}$$

From Compression $C$,

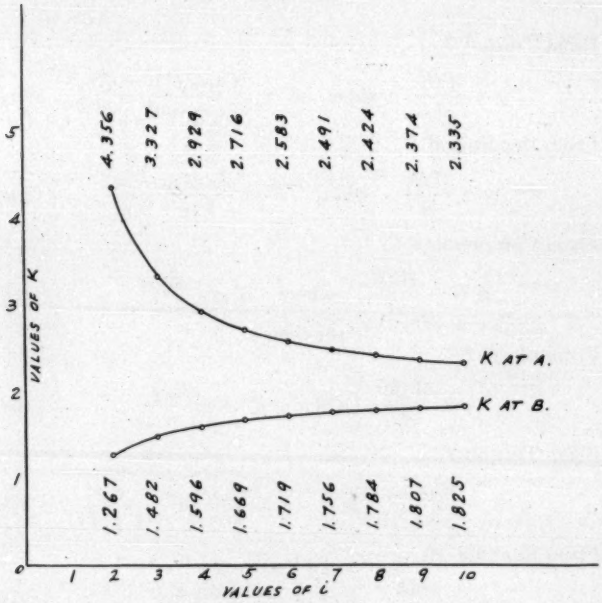
$$p = -\frac{WiK}{A}, \quad \text{where} \quad K = \frac{\sin \phi}{k(i+1)}. \tag{14b}$$

The combined stress at points $A$ and $B$ can be found by adding algebraically like stresses then applying the formula

$$f = \frac{p}{2} + \left(\frac{p^2}{4} + q^2\right)^{1/2}, \tag{28}$$

which gives combined direct stress $f$ for component direct stress $p$ and shear stress $q$.

FIG. 7.




$$f = \frac{W i}{A} \cdot K$$

Combined stress.

Figure 7 shows graphically also tabulated values of $K$ for use in the combined stress formula

$$f = \frac{WiK}{A} . \qquad (29)$$

If the component stress formulas were combined algebraically to give $f_A$ and $f_B$ the results would be too complicated for use, however $K$ as used in $f = WiK/A$ can be expressed by the empirical forms:

$$K = 2\left(\frac{i + .75}{i - .75}\right) \quad \text{(at point } A\text{)},$$

$$K = 2\left(\frac{i - .45}{i + .45}\right) \quad \text{(at point } B\text{)}.$$

These forms give the values of $K$ to within one per cent. of the computed values in figure 7.

Thus the combined stress formulas can be written:

$$\text{Combined stress at } A, \quad f_A = \frac{2Wi}{A}\left(\frac{i + .75}{i - .75}\right), \qquad (30)$$

$$\text{Combined stress at } B, \quad f_B = \frac{2Wi}{A}\left(\frac{i - .45}{i + .45}\right). \qquad (31)$$

In the use of above formula together with graph or table for values of $K$ it must be noted that the stress is correct only for a spring compressed solid where $\tan \phi = 1/\pi i$. However in heavy helical springs the total deflection per coil is usually small so that there is little change in the stress due to the variable $\phi$ when the spring is only partially loaded and the adjacent coils are not in contact.

By referring to figure 7 it will be noted that the value of $K$ at points $A$ and $B$ verge towards a value 2 which is the constant used in formula (5). However $K$ will only approximate the value 2 for very large values of $i$, that is much larger than experienced in practice, and for small values of $i$ the stress at point $A$ is very much larger than given by the common formula (5).

It is unfortunate that the maximum combined stress occurs at point $A$ which is inside the bar and therefore not easily observed so that when a fracture develops at this point it is not noticed until complete failure occurs.

That the maximum combined stress occurs at a point only makes it difficult to detect over stress from a static spring test. A very small area may be stressed to the elastic limit but the weakness will not show up in a static test as the large area stressed elastically will cause the spring to return, when unloaded, to the original free height yet when in service it will fatigue rapidly.

# NOTE ON A METHOD OF MEASURING CATHODE RAY OSCILLOGRAPH FIGURES.

BY

RAYMOND M. WILMOTTE, M.A.

Aircraft Radio Corporation, Boonton, N. J.

The most versatile of electrical instruments, the cathode ray oscillograph, has always suffered from a serious limitation, namely its lack of accuracy. This is particularly so, as might be expected, in the more sensitive instruments, or the low voltage types.

Several factors effect the linearity of the calibration of the tube.

Firstly, it is usually necessary to apply a magnetic field in order that the undeflected ray may impinge on some suitable point of the screen. With due care it is possible to arrange that this field be sensibly uniform, in which case the electron ray within the tube is not likely to be distorted. Such conditions are not usually very easy to obtain, especially if the tube has to be magnetically screened in order to guard against distortion caused by unknown, extraneous magnetic fields.

Secondly, the screen is usually laid on a glass surface which should be at all points normal to the impinging ray. This is seldom the case and errors in calibration are very likely to arise owing to mechanical distortions in the glass supporting the screen.

Thirdly, the deflecting elements of the tubes are often inaccurately adjusted, so that the deflecting axes are either not quite at right angles to each other or the values of the applied deflecting fields causing unit deflection are not equal along the two axes. This latter error is usually present because the distances of the deflecting plates from the screen are not equal.

Fourthly, the image on the screen is reproduced either by drawing, which is of course very inaccurate, or by photography, in which case the axes are usually not marked on the photograph. In the case of a photograph, it is possible to apply a series of D.C. voltages on the deflecting plates and

superimpose on the photographic plate a number of parallel lines similar to graph paper.

Fifthly, the calibration depends on the accelerating voltage applied to the anode. If this were unknowingly altered an error would ensue.

The method to be described automatically eliminates all the above causes of error. It is however, applicable, only to the case when an apparently stationary image is produced on the screen.

The method has been employed in an experiment in which the time scale was produced by means of a rotating potentiometer. It was desired to measure the maximum voltage of a recurring wave form of very irregular shape. During every other rotation of the potentiometer brush, the deflecting plate of the oscillograph was disconnected from the apparatus producing the wave form under examination and, instead, was connected to a D.C. voltage of known value. During alternate revolutions of the potentiometer then, the image produced on the screen was that due to the wave form under examination while, during the other revolutions, the image of the screen was a simple straight line the position of which depended on the value of the D.C. voltage applied to the deflecting plate. Owing to the phenomenon of the persistence of vision the resulting image on the screen as seen by the observer consisted of the wave form under examination and a straight line. By the simple process of adjusting the D.C. voltage applied to the deflecting plate the straight line could be moved up or down. It was adjusted to be a tangent to the highest point of the wave form curve. The D.C. voltage was ascertained and was taken to be equal to the maximum voltage of the wave form.

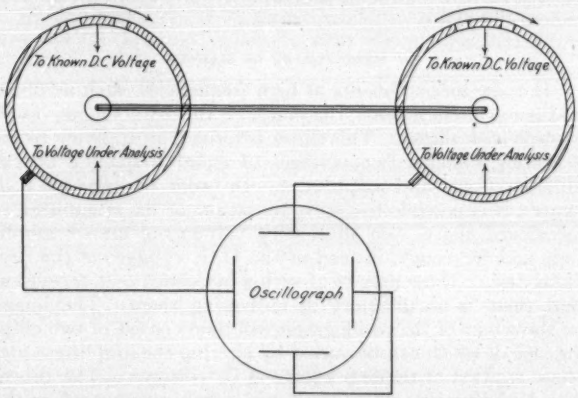
This experiment was actually carried out under very disadvantageous conditions in which the visual methods would have failed to give satisfactory accuracy. The cathode ray tube employed was of very rough construction, the glass of the tube was very uneven and electrical machines in the neighbourhood were continually causing variable magnetic conditions within the tube. These causes made the line produced by the D.C. voltage far from straight and the positions of the kinks in it varied with the value of the D.C. deflecting voltage,

thus giving an idea of the errors which would have been involved had the method described not been used.

Extensions of this method are obvious and will now be briefly considered.

#### (a) REPRODUCTION OF THE WHOLE IMAGE.

Each point of a curve on the screen of a cathode ray oscillograph corresponds to two coördinates of deflecting voltages, proportional to the instantaneous values of the voltages applied to the two deflecting systems at the corresponding instant. In order to find these electrical coördinates refering to any given point on the curve, a commutator may be inserted in each of the leads connecting the two deflecting systems of the




oscillograph. These commutators (see diagram), for a short period during each revolution, break the connection to the source of potential which is being analysed and substitute at the same instant known D.C. voltages. The result on the screen is an image of the wave form under analysis and an isolated spot, which spot can be varied in position by altering the values of the two D.C. voltages. The procedure to reproduce a wave form, then, is to adjust the position of the isolated spot so as to superimpose it on the curve. The read-

ing of the D.C. voltage at any instant will give the coördinates of point on the curve. The isolated spot is then made to follow the curve and the coördinates of as many points on the curve as is considered desirable are thus measured. The most convenient method of varying the D.C. voltages is by means of two potentiometers, one for each applied D.C. voltage. For convenience it is possible to connect the potentiometers mechanically to a pencil and draw the curve directly on the paper as the isolated spot is made to follow the curve. In many experiments one axis corresponds to a time scale and this time scale instead of being linear is often conveniently made sinusoidal. In such a case, the mechanical coupling of one potentiometer to the pencil may be designed so that the sinusoidal time scale on the oscillograph is translated into a linear one on the paper.

(b) MEASUREMENT OF ELLIPSES.

In some measurements at high frequencies, such as phase and power mesurements, the image on the cathode ray oscillograph is an ellipse. This ellipse is formed by applying to the deflecting systems two voltages of equal frequency but of different phase and amplitudes. In order to measure such figures, it is possible to apply, by means of the commutators mentioned above, two alternating voltages of known amplitude and frequency, instead of the D.C. voltages of the previous case. These may be of such a frequency (e.g. 60 cycles) that there is no difficulty in measuring them. The image on the screen of the oscillograph will then consist of two ellipses, one of which can be varied by altering the amplitude and phase relation of the two known A.C. voltages. The procedure is to alter this ellipse until it is exactly superposed on the other.

(c) APPLICATION TO VERY HIGH FREQUENCIES.

In low voltage cathode ray oscillographs the time the electrons take to traverse the deflecting field is not always negligible and at very high frequencies a considerable error is liable to be present. It is possible to produce with a fair degree of accuracy high frequency potentials of known value by passing a known current through a non-inductive resistance. The procedure in analysing a figure on a cathode ray oscillograph

produced by very high frequency potentials would be to apply to the deflecting systems of the oscillograph, via the commutators, known voltages of frequencies approximately equal to those of the voltages under analysis. The figure seen on the screen of the oscillograph will then consist of the figure under analysis and a straight line passing through the origin. (If the known voltages are not exactly in phase this line will not be straight, but will appear as a narrow ellipse.) The end of this straight line corresponds to the isolated point which was obtained by the D.C. voltages in case (*a*) above. By varying the known high frequency voltages, this straight line can be made to change. The procedure is to alter this line so that its end point follows the curve under analysis, in the same way as the isolated spot was moved in case (*a*).

In conclusion I wish to thank Mr. C. A. Luxford who constructed the original apparatus with which the method was tested.

**A Factor in the Temperature of the Stratosphere.** W. J. HUMPHREYS. (*Journal Maryland Academy of Sciences*, Jan., 1930.) The temperature of the air decreases with elevation above the surface of the earth until the upper limit of the troposphere is reached, that is, up to the height at which convection currents cease. At middle latitudes this height is from 6 to 7 miles and in tropical regions 8 to 10 miles. Above the troposphere is the stratosphere. Numerous records coming from balloons carrying registering instruments have shown that as soon as the troposphere has been left below the temperature of the air is no longer dependent on the height, nor does it change from day to night to any great extent. In addition the stratosphere is coldest over the equator and grows warmer with increasing latitude. Along with this strange distribution of temperature is the equally strange fact, discovered not long ago, that over the equator where the sunshine is strongest there is less ozone than in higher latitudes. This ozone is situated above the greatest elevation reached by balloons. Ozone powerfully absorbs the radiation coming from the earth and its atmosphere, and, since it does not continue to grow warmer and warmer, reradiates the absorbed energy. Part is sent upward and part downward. The part radiated downward helps to raise the temperature of the air below the ozone. Where there is least ozone the least radiation is absorbed and the least reradiated downward. This is the case at the equator. "In short, one reason (not the only one) why the stratospere becomes colder and colder as we go from high latitudes to the equator, is because the ozone blanket at the same time grows thinner. It is a little like sleeping warmly outdoors under a quilt or shivering under a sheet—lots depends on the kind and quantity of covering one has."

G. F. S.

# DYNAMICAL ANALYSIS OF MACHINES.*

BY

R. EKSERGIAN, M.S., M.E.E., Ph.D.,

Consultant, Engineering Department,
E. I. DuPont de Nemours & Company,
Member of the Institute.

## SYSTEMS WITH TWO OR MORE DEGREES OF FREEDOM.

The use of the kinematic procedure of velocity ratios and their derivatives may be extended to systems with several degrees of freedom, provided the velocity ratios be regarded as partial with respect to any given coördinate. That is, by maintaining the other coördinates constant, we reduce the system to one degree of freedom from which the partial velocity ratios may be obtained corresponding to the ordinary velocity ratios for a system with one degree of freedom.

The partial velocity ratios themselves, however, are in general functions of the given and remaining coördinates and we find the reciprocal relation, that the partial derivative of a partial velocity ratio of the given coördinate with respect to any other coördinate is equal to the partial derivative of the partial velocity ratio of any other coördinate with respect to the given coördinate.

We will proceed now to form the equations of motion corresponding to the several degrees of freedom in terms of the partial velocity ratios. From D'Alembert's equation of Virtual Work,

$$\Sigma m \frac{d^2x}{dt^2}\delta x + \Sigma m \frac{d^2y}{dt^2}\delta y + \Sigma I \frac{d^2\phi}{dt^2}\delta\phi = \Sigma X\delta x + \Sigma Y\delta y + \Sigma L\delta\phi,$$

where $m$ is the mass and $I$ the moment of inertia of a member and $X$, $Y$, and $L$ are the applied work forces and moments, the virtual work of the reactions of constraint being nil.

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 667, vol. 209, May, 1930.

If $x$, $y$ and $\phi$ are functions of the independent coördinates, $\alpha$, $\beta$, $\gamma$, etc., then,

$$\delta x = \frac{\partial x}{\partial \alpha}\delta\alpha + \frac{\partial x}{\partial \beta}\delta\beta + \frac{\partial x}{\partial \gamma}\delta\gamma \cdots,$$

$$\delta y = \frac{\partial y}{\partial \alpha}\delta\alpha + \frac{\partial y}{\partial \beta}\delta\beta + \frac{\partial \gamma}{\partial y}\delta\gamma \cdots,$$

$$\delta\phi = \frac{\partial \phi}{\partial \alpha}\delta\alpha + \frac{\partial \phi}{\partial \beta}\delta\beta + \frac{\partial \phi}{\partial \gamma}\delta\gamma \cdots,$$

and the velocities are,

$$\dot{x} = \frac{\partial x}{\partial \alpha}\dot{\alpha} + \frac{\partial x}{\partial \beta}\dot{\beta} + \frac{\partial x}{\partial \gamma}\dot{\gamma} \cdots,$$

$$\dot{y} = \frac{\partial y}{\partial \alpha}\dot{\alpha} + \frac{\partial y}{\partial \beta}\dot{\beta} + \frac{\partial y}{\partial \gamma}\dot{\gamma} \cdots,$$

$$\dot{\phi} = \frac{\partial \phi}{\partial \alpha}\dot{\alpha} + \frac{\partial \phi}{\partial \beta}\dot{\beta} + \frac{\partial \phi}{\partial \gamma}\dot{\gamma} \cdots,$$

now $\frac{\partial x}{\partial \alpha}, \frac{\partial x}{\partial \beta}$, etc., are functions $\alpha$, $\beta$, $\gamma$, $\cdots$—so that

$$\frac{d}{dt}\left(\frac{\partial x}{\partial \alpha}\dot{\alpha}\right) = \frac{\partial x}{\partial \alpha}\ddot{\alpha} + \frac{d}{dt}\left(\frac{\partial x}{\partial \alpha}\right)\dot{\alpha}$$

$$= \frac{\partial x}{\partial \alpha}\ddot{\alpha} + \frac{\partial^2 x}{\partial \alpha^2}\dot{\alpha}^2 + \frac{\partial^2 x}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta} + \frac{\partial^2 x}{\partial \alpha \partial \gamma}\dot{\gamma}\dot{\alpha},$$

$$\frac{d}{dt}\left(\frac{\partial x}{\partial \beta}\dot{\beta}\right) = \frac{\partial x}{\partial \beta}\ddot{\beta} + \frac{d}{dt}\left(\frac{\partial x}{\partial \beta}\right)\dot{\beta}$$

$$= \frac{\partial x}{\partial \beta}\ddot{\beta} + \frac{\partial^2 x}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta} + \frac{\partial^2 x}{\partial \beta^2}\dot{\beta}^2 + \frac{\partial^2 x}{\partial \beta \partial \gamma}\dot{\gamma}\dot{\beta}.$$

Therefore,

$$\ddot{x} = \frac{\partial x}{\partial \alpha}\ddot{\alpha} + \frac{\partial x}{\partial \beta}\ddot{\beta} + \frac{\partial x}{\partial \gamma}\ddot{\gamma} \cdots$$

$$+ \frac{\partial^2 x}{\partial \alpha^2}\dot{\alpha}^2 + \frac{\partial^2 x}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta} + \frac{\partial^2 x}{\partial \alpha \partial \gamma}\dot{\gamma}\dot{\alpha} \cdots$$

$$+ \frac{\partial^2 x}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta} + \frac{\partial^2 x}{\partial \beta^2}\dot{\beta}^2 + \frac{\partial^2 x}{\partial \beta \partial \gamma}\dot{\gamma}\dot{\beta} \cdots$$

$$+ \frac{\partial^2 x}{\partial \alpha \partial \gamma}\dot{\alpha}\dot{\gamma} + \frac{\partial^2 x}{\partial \beta \partial \gamma}\dot{\beta}\dot{\gamma} + \frac{\partial^2 x}{\partial \gamma^2}\dot{\gamma}^2 \cdots.$$

The partial velocity ratios are,

$$K_{x\alpha}=\frac{\partial x}{\partial \alpha},\qquad K_{x\beta}=\frac{\partial x}{\partial \beta},\qquad K_{x\gamma}=\frac{\partial x}{\partial \gamma},$$

$$K_{y\alpha}=\frac{\partial y}{\partial \alpha},\qquad K_{y\beta}=\frac{\partial y}{\partial \beta},\qquad K_{y\gamma}=\frac{\partial y}{\partial \gamma},$$

$$K_{\phi\alpha}=\frac{\partial \phi}{\partial \beta},\qquad K_{\phi\beta}=\frac{\partial \phi}{\partial \alpha},\qquad K_{\phi\gamma}=\frac{\partial \phi}{\partial \gamma},\qquad \text{etc.}$$

Also we note

$$\frac{\partial K_{x\alpha}}{\partial \beta}=\frac{\partial K_{x\beta}}{\partial \alpha}=\frac{\partial^2 x}{\partial \alpha \partial \beta},\qquad \frac{\partial K_{x\beta}}{\partial \gamma}=\frac{\partial K_{x\gamma}}{\partial \gamma}=\frac{\partial^2 x}{\partial \beta \partial \gamma},$$

$$\frac{\partial K_{x\gamma}}{\partial \alpha}=\frac{\partial K_{x\alpha}}{\partial \gamma}=\frac{\partial^2 x}{\partial \alpha \partial \gamma},\qquad \text{etc.}$$

Since $\alpha$, $\beta$, $\gamma$, etc., are independent coördinates, the equations of motion take the form,

$$\begin{aligned}
&\Sigma m\Big[ K_{x\alpha}{}^2\ddot{\alpha} + K_{x\alpha}K_{x\beta}\ddot{\beta} + K_{x\alpha}K_{x\gamma}\ddot{\gamma} + K_{x\alpha}\frac{\partial K_{x\alpha}}{\partial \alpha}\dot{\alpha}^2 \\
&+ K_{x\alpha}\frac{\partial K_{x\beta}}{\partial \beta}\dot{\beta}^2 + K_{x\alpha}\frac{\partial K_{x\gamma}}{\partial \gamma}\dot{\gamma}^2 + 2K_{x\alpha}\frac{\partial K_{x\alpha}}{\partial \beta}\dot{\alpha}\dot{\beta} \\
&+ 2K_{x\alpha}\frac{\partial K_{x\beta}}{\partial \gamma}\dot{\beta}\dot{\gamma} + 2K_{x\alpha}\frac{\partial K_{x\gamma}}{\partial \alpha}\dot{\gamma}\dot{\alpha}\Big] + \Sigma m\Big[ K_{y\alpha}{}^2\ddot{\alpha} \\
&+ K_{y\alpha}K_{y\beta}\ddot{\beta} + K_{y\alpha}K_{y\gamma}\ddot{\gamma} + K_{y\alpha}\frac{\partial K_{y\alpha}}{\partial \alpha}\dot{\alpha}^2 + K_{y\alpha}\frac{\partial K_{y\beta}}{\partial \beta}\cdot\dot{\beta}^2 \\
&+ K_{y\alpha}\frac{\partial K_{y\gamma}}{\partial \gamma}\dot{\gamma}^2 + 2K_{y\alpha}\frac{\partial K_{y\alpha}}{\partial \beta}\dot{\alpha}\dot{\beta} + 2K_{y\alpha}\frac{\partial K_{y\beta}}{\partial \gamma}\dot{\beta}\dot{\gamma} \\
&+ 2K_{x\alpha}\frac{\partial K_{x\gamma}}{\partial \alpha}\dot{\gamma}\dot{\alpha}\Big] + \Sigma TI\Big[ K_{\phi\alpha}{}^2\ddot{\alpha} + K_{\phi\alpha}K_{\phi\beta}\ddot{\beta} + K_{\phi\alpha}K_{\phi\gamma}\ddot{\gamma} \\
&+ K_{\phi\alpha}\frac{\partial K_{\phi\alpha}}{\partial \gamma}\cdot\dot{\alpha}^2 + K_{\phi\alpha}\frac{\partial K_{\phi\beta}}{\partial \beta}\cdot\dot{\beta}^2 + K_{\phi\alpha}\frac{\partial K_{\phi\gamma}}{\partial \gamma}\dot{\gamma}^2 \\
&+ 2K_{\phi\alpha}\frac{\partial K_{\phi\alpha}}{\partial \beta}\dot{\alpha}\dot{\beta} + 2K_{\phi\alpha}\frac{\partial K_{\phi\beta}}{\partial \gamma}\dot{\beta}\dot{\gamma} + 2K_{\phi\alpha}\frac{\partial K_{\phi\gamma}}{\partial \alpha}\dot{\gamma}\dot{\alpha}\Big] \\
&\qquad = \Sigma XK_{x\alpha} + \Sigma YK_{y\alpha} + \Sigma LK_{\phi\alpha} - \frac{\partial V}{\partial \alpha}
\end{aligned}$$

with similar symmetrical equations for the coördinates $\beta$, $\gamma$, etc.

For the more common case of only two degrees of freedom we have,

$$\Sigma m\left[K_{x\alpha}{}^2\ddot{\alpha}+K_{x\alpha}K_{x\beta}\ddot{\beta}+K_{x\alpha}\frac{\partial K_{x\alpha}}{\partial\alpha}\dot{\alpha}^2+K_{x\alpha}\frac{\partial K_{x\beta}}{\partial\beta}\cdot\dot{\beta}^2+2K_{x\alpha}\frac{\partial K_{x\alpha}}{\partial\beta}\dot{\alpha}\dot{\beta}\right]+\Sigma m\left[K_{y\alpha}{}^2\ddot{\alpha}+K_{y\alpha}K_{y\beta}\ddot{\beta}+K_{y\alpha}\frac{\partial K_{y\alpha}}{\partial\alpha}\dot{\alpha}^2+K_{y\alpha}\frac{\partial K_{y\beta}}{\partial\beta}\cdot\dot{\beta}^2+2K_{y\alpha}\frac{\partial K_{y\alpha}}{\partial\beta}\dot{\alpha}\dot{\beta}\right]+\Sigma TI\left[K_{\phi\alpha}{}^2\ddot{\alpha}+K_{\phi\alpha}K_{\phi\beta}\ddot{\beta}+K_{\phi\alpha}\frac{\partial K_{\phi\alpha}}{\alpha\partial}\dot{\alpha}^2+K_{\phi\alpha}\frac{\partial K_{\phi\beta}}{\partial\beta}\cdot\dot{\beta}^2+2K_{\phi\alpha}\frac{\partial K_{\phi\alpha}}{\partial\beta}\dot{\alpha}\dot{\beta}\right]=\Sigma XK_{x\alpha}+\Sigma YK_{y\alpha}+\Sigma LK_{\phi\alpha}-\frac{\partial V}{\partial\alpha}$$

with a similar equation for $\beta$.

The equations can be verified by a substitution of the partial velocity ratios in Lagrange's equations. For simplicity we will consider the derivation for only two degrees, but the procedure can be extended to any number of degrees of freedom. Moreover we will designate any displacement as $\delta x$, $\delta y$, or $\delta\phi$ by $\delta s$. The reduced forces with respect to the coördinates $\alpha$ and $\beta$ are $\Phi_\alpha$ and $\Phi_\beta$ respectively. If $T$ is the kinetic energy of the system, then from Lagrange's equation

$$\Phi_\alpha=\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\alpha}}\right)-\frac{\partial T}{\partial\alpha}=\Sigma\left(m\frac{d^2s}{dt^2}\frac{\partial s}{\partial\alpha}\right),$$

now

$$T=\Sigma\frac{m}{2}\left(\frac{\partial s}{\partial\alpha}\dot{\alpha}+\frac{\partial s}{\partial\beta}\dot{\beta}\right)^2=\Sigma\frac{m}{2}\left[\left(\frac{\partial s}{\partial\alpha}\right)^2\dot{\alpha}^2+\left(\frac{\partial s}{\partial\beta}\right)^2\dot{\beta}^2+2\frac{\partial s}{\partial\beta}\frac{\partial s}{\partial\alpha}\dot{\beta}\dot{\alpha}\right],$$

$$\frac{\partial T}{\partial\dot{\alpha}}=\Sigma m\left[\left(\frac{\partial s}{\partial\alpha}\right)^2\dot{\alpha}^2+\frac{\partial s}{\partial\alpha}\frac{\partial s}{\partial\beta}\dot{\beta}\right];$$

$$\therefore \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) = \Sigma m\left[\left(\frac{\partial s}{\partial \alpha}\right)^2 \ddot{\alpha} + \frac{\partial s}{\partial \alpha}\frac{\partial s}{\partial \beta}\ddot{\beta} + \dot{\alpha}\frac{d}{dt}\left(\frac{\partial s}{\partial \alpha}\right)^2 + \dot{\beta}\frac{d}{dt}\left(\frac{\partial s}{\partial \alpha}\frac{\partial s}{\partial \beta}\right)\right],$$

$$\frac{d}{dt}\left(\frac{\partial s}{\partial \alpha}\right)^2 = 2\frac{\partial s}{\partial \alpha}\frac{d}{dt}\left(\frac{\partial s}{\partial \alpha}\right) = 2\frac{\partial s}{\partial \alpha}\left[\frac{\partial^2 s}{\partial \alpha^2}\dot{\alpha} + \frac{\partial^2 s}{\partial \alpha \partial \beta}\dot{\beta}\right]$$

$$\frac{d}{dt}\left(\frac{\partial s}{\partial \alpha}\frac{\partial s}{\partial \beta}\right) = \frac{\partial s}{\partial \alpha}\frac{d}{dt}\left(\frac{\partial s}{\partial \beta}\right) + \frac{\partial s}{\partial \beta}\frac{d}{dt}\left(\frac{\partial s}{\partial \alpha}\right)$$

$$= \frac{\partial s}{\partial \alpha}\left[\frac{\partial^2 s}{\partial \alpha \partial \beta}\dot{\alpha} + \frac{\partial^2 s}{\partial \beta^2}\dot{\beta}\right] + \frac{\partial s}{\partial \beta}\left[\frac{\partial^2 s}{\partial \alpha^2}\dot{\alpha} + \frac{\partial^2 s}{\partial \alpha \partial \beta}\dot{\beta}\right],$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) = \Sigma m\left[\left(\frac{\partial s}{\partial \alpha}\right)^2 \ddot{\alpha} + \frac{\partial s}{\partial \alpha}\frac{\partial s}{\partial \beta}\ddot{\beta} + 2\dot{\alpha}^2\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha^2} + 2\dot{\alpha}\dot{\beta}\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha \partial \beta} + \dot{\beta}\dot{\alpha}\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha \partial \beta} + \dot{\beta}^2\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \beta^2} + \dot{\beta}\dot{\alpha}\frac{\partial s}{\partial \beta}\frac{\partial^2 s}{\partial \alpha^2} + \dot{\beta}^2\frac{\partial s}{\partial \beta}\frac{\partial^2 s}{\partial \alpha \partial \beta}\right]$$

$$= \Sigma m\left[\left(\frac{\partial s}{\partial \alpha}\right)^2 \ddot{\alpha} + \frac{\partial s}{\partial \alpha}\frac{\partial s}{\partial \beta}\ddot{\beta} + 2\dot{\alpha}^2\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha^2} + 3\dot{\alpha}\dot{\beta}\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha \partial \beta} + \dot{\alpha}\frac{\partial s}{\partial \beta}\frac{\partial^2 s}{\partial \alpha^2} + \dot{\beta}^2\frac{\partial s}{\partial \beta}\frac{\partial^2 s}{\partial \alpha \partial \beta} + \dot{\beta}^2\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \beta^2}\right],$$

$$\frac{\partial T}{\partial \alpha} = \Sigma m\left[\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha^2}\dot{\alpha}^2 + \frac{\partial s}{\partial \beta}\frac{\partial^2 s}{\partial \alpha \partial \beta}\dot{\beta}^2 + \frac{\partial s}{\partial \beta}\frac{\partial^2 s}{\partial \alpha^2}\dot{\alpha}\dot{\beta} + \frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta}\right]$$

$$\therefore \quad \Phi_\alpha = \Sigma m\left[\left(\frac{\partial s}{\partial \alpha}\right)^2 \ddot{\alpha} + \frac{\partial s}{\partial \alpha}\frac{\partial s}{\partial \beta}\ddot{\beta} + \frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha^2}\dot{\alpha}^2 + 2\frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta} + \frac{\partial s}{\partial \alpha}\frac{\partial^2 s}{\partial \beta^2}\dot{\beta}^2\right].$$

Substituting the partial velocity rations, $K_{s\alpha} = \frac{\partial s}{\partial \alpha}$, $K_{s\beta} = \frac{\partial s}{\partial \beta}$

and noting $\frac{\partial K_{z\alpha}}{\partial \beta} = \frac{\partial K_{z\beta}}{\partial \alpha}$ we have

$$\Phi_\alpha = \sum m\Big[ K_{z\alpha}\ddot{\alpha}^2 + K_{z\alpha}K_{z\beta}\ddot{\beta} + K_{z\alpha}\frac{\partial K_{z\alpha}}{\partial \alpha}\dot{\alpha}^2 + 2K_{z\alpha}\frac{\partial K_{z\beta}}{\partial \beta}\dot{\alpha}\dot{\beta} + K_{z\alpha}\frac{\partial K_{z\beta}}{\partial \beta}\dot{\beta}^2\Big]$$

with a similar equation for $\Phi_\beta$.

### THE EQUATIONS OF MOMENTUM.

We will now consider the equations for the rate of change of the linear and angular momentum for systems of two or more degrees of freedom, in terms of the partial velocity ratios and their partial derivatives.

The equations for the rate of change of momentum are,

$$\sum X = \sum m\ddot{x}, \qquad \sum Y = \sum m\ddot{y},$$
$$\sum L = \sum(Yx - Xy) = \sum I\ddot{\phi} + \sum M(\ddot{y}x - \ddot{x}y),$$

where the general accelerations may be expressed in terms of the partial velocity ratios and their partial derivatives. The accelerations are,

$$\begin{aligned}
\ddot{x} &= K_{x\alpha}\ddot{\alpha} + K_{x\beta}\ddot{\beta} + K_{x\gamma}\ddot{\gamma} + \cdots \\
&\quad + \frac{\partial K_{x\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{x\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta} + \frac{\partial K_{x\gamma}}{\partial \alpha}\dot{\alpha}\dot{\gamma} \cdots \\
&\quad + \frac{\partial K_{x\alpha}}{\partial \beta}\dot{\beta}\dot{\alpha} + \frac{\partial K_{x\beta}}{\partial \beta}\dot{\beta}^2 + \frac{\partial K_{x\gamma}}{\partial \beta}\dot{\beta}\dot{\gamma} \cdots \\
&\quad + \frac{\partial K_{x\alpha}}{\partial \gamma}\dot{\gamma}\dot{\alpha} + \frac{\partial K_{x\beta}}{\partial \gamma}\dot{\gamma}\dot{\beta} + \frac{\partial K_{x\gamma}}{\partial \gamma}\dot{\gamma}^2 \cdots \\
&\quad \cdots\cdots\cdots\

$$\ddot{\phi} = K_{\phi\alpha}\ddot{\alpha} + K_{\phi\beta}\ddot{\beta} + K_{\phi\gamma}\ddot{\gamma} \cdots$$
$$+ \frac{\partial K_{\phi\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{\phi\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta} + \frac{\partial K_{\phi\gamma}}{\partial \alpha}\dot{\alpha}\dot{\gamma} \cdots$$
$$+ \frac{\partial K_{\phi\alpha}}{\partial \beta}\dot{\beta}\dot{\alpha} + \frac{\partial K_{\phi\beta}}{\partial \beta}\dot{\beta}^2 + \frac{\partial K_{\phi\gamma}}{\partial \beta}\dot{\beta}\dot{\gamma} \cdots$$
$$+ \frac{\partial K_{\phi\alpha}}{\partial \gamma}\dot{\gamma}\dot{\alpha} + \frac{\partial K_{\phi\beta}}{\partial \gamma}\dot{\gamma}\dot{\beta} + \frac{\partial K_{\gamma\phi}}{\partial \gamma}\dot{\gamma}^2 \cdots,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$

Due to the reciprocal relations,

$$\frac{\partial K_{x\alpha}}{\partial \beta} = \frac{\partial K_{x\beta}}{\partial \alpha}, \qquad \frac{\partial K_{x\beta}}{\partial \gamma} = \frac{\partial K_{x\gamma}}{\partial \beta}, \qquad \frac{\partial K_{x\gamma}}{\partial \alpha} = \frac{\partial K_{x\alpha}}{\partial \gamma}, \qquad \text{etc.}$$

The accelerations reduce to

$$\ddot{x} = K_{x\alpha}\ddot{\alpha} + K_{x\beta}\ddot{\beta} + K_{x\gamma}\ddot{\gamma} + \frac{\partial K_{x\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{x\beta}}{\partial \beta}\dot{\beta}^2 + \frac{\partial K_{x\gamma}}{\partial \gamma}\dot{\gamma}^2 \cdots$$
$$+ 2\frac{\partial K_{x\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta} + 2\frac{\partial K_{x\gamma}}{\partial \beta}\dot{\beta}\dot{\gamma} + 2\frac{\partial K_{x\alpha}}{\partial \gamma}\dot{\gamma}\dot{\alpha} \cdots,$$

$$\ddot{y} = K_{y\alpha}\ddot{\alpha} + K_{y\beta}\ddot{\beta} + K_{y\gamma}\ddot{\gamma} + \frac{\partial K_{y\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{y\beta}}{\partial \beta}\dot{\beta}^2 + \frac{\partial K_{y\gamma}}{\partial \gamma}\dot{\gamma}^2 \cdots$$
$$+ 2\frac{\partial K_{y\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta} + 2\frac{\partial K_{y\gamma}}{\partial \beta}\dot{\beta}\dot{\gamma} + 2\frac{\partial K_{y\alpha}}{\partial \gamma}\dot{\gamma}\dot{\alpha} \cdots,$$

$$\ddot{\phi} = K_{\phi\alpha}\ddot{\alpha} + K_{\phi\beta}\ddot{\beta} + K_{\phi\gamma}\ddot{\gamma} + \frac{\partial K_{\phi\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{\phi\beta}}{\partial \beta}\dot{\beta}^2 + \frac{\partial K_{\phi\gamma}}{\partial \gamma}\dot{\gamma}^2 \cdots$$
$$+ 2\frac{\partial K_{\phi\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta} + 2\frac{\partial K_{\phi\gamma}}{\partial \beta}\dot{\beta}\dot{\gamma} + 2\frac{\partial K_{\phi\alpha}}{\partial \gamma}\dot{\gamma}\dot{\alpha} \cdots.$$

We may derive similar equations for the coördinates, etc. The forces $\sum X$, $\sum Y$ and $\sum L$ include all external forces, including the reactions of external constraints, for the system under consideration. The internal reactions due to their mutual nature disappear.

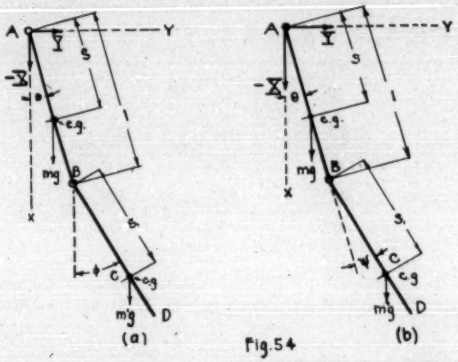



Fig. 54

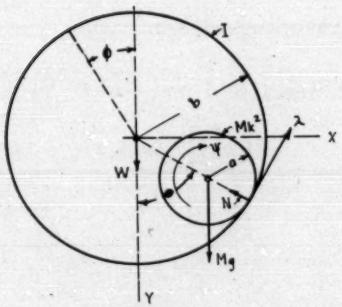



Fig. 55

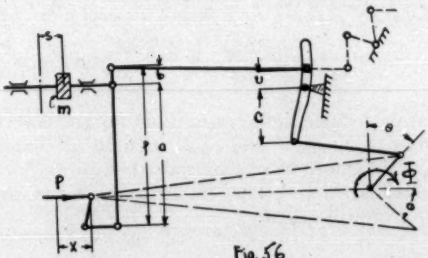



Fig. 56

For two degrees of freedom the equations of momentum, are,

$$\Sigma X = \Sigma m\left(K_{x\alpha}\ddot{\alpha} + K_{x\beta}\beta + \frac{\partial K_{x\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{x\beta}}{\partial \beta}\dot{\beta}^2 + 2\frac{\partial K_{x\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta}\right),$$

$$\Sigma Y = \Sigma m\left(K_{y\alpha}\ddot{\alpha} + K_{y\beta}\ddot{\beta} + \frac{\partial K_{y\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{y\beta}}{\partial \beta}\dot{\beta}^2 + 2\frac{\partial K_{y\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta}\right),$$

$$\Sigma L = \Sigma I\left(K_{\phi\alpha}\ddot{\alpha} + K_{\phi\beta}\ddot{\beta} + \frac{\partial K_{\phi\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{\phi\beta}}{\partial \beta}\dot{\beta}^2 + 2\frac{\partial K_{\phi\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta}\right)$$
$$+ \Sigma m\left[\left(K_{y\alpha}\ddot{\alpha} + K_{y\beta}\beta + \frac{\partial K_{y\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{y\beta}}{\partial \beta}\dot{\beta}^2 + 2\frac{\partial K_{y\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta}\right)x\right.$$
$$\left. - \left(K_{x\alpha}\ddot{\alpha} + K_{x\beta}\ddot{\beta} + \frac{\partial K_{x\alpha}}{\partial \alpha}\dot{\alpha}^2 + \frac{\partial K_{x\beta}}{\partial \beta}\dot{\beta}^2 + 2\frac{\partial K_{x\beta}}{\partial \alpha}\dot{\alpha}\dot{\beta}\right)y\right],$$

where as before it is to be noted,

$$\frac{\partial K_{x\beta}}{\partial \alpha} = \frac{\partial K_{x\alpha}}{\partial \beta}, \qquad \frac{\partial K_{y\beta}}{\partial \alpha} = \frac{\partial K_{y\alpha}}{\partial \beta}, \qquad \frac{\partial K_{\phi\beta}}{\partial \alpha} = \frac{\partial K_{\phi\alpha}}{\partial \beta}.$$

THE DOUBLE PENDULUM FIGS. (54a AND b).

As an example of systems of two or more degrees of freedom we will consider some simple applications as the equations for the double pendulum derived from their partial velocity ratios.

Let $\theta$ and $\phi$, Fig. (54*a*), be the angular coördinates of the rods $AB$ and $BD$ with respect to the vertical, where the length of rod $AB = l$, and its c. of g. is at a distance $s$ from $A$, while the c. of g. of rod $BD$ is at a distance $s_1$ from the joint $B$. Let $I$ = moment of inertia of $AB$ about $A$ and $m$ its mass and $m'$ and $I_g'$ be the mass and moment of inertia of $BD$ with respect to its c. of g. at $C$.

The coördinates of $C$ and rod $BD$ are

$$x_c = l\cos\theta + s_1\cos\theta,$$
$$y_c = l\sin\theta + s_1\sin\theta,$$

so that the partial velocity ratios are,

$$K_{x\theta} = -l\sin\theta, \qquad K_{x\phi} = -s_1\sin\theta,$$
$$K_{y\theta} = l\cos\theta, \qquad K_{y\phi} = s_1\cos\theta.$$

The velocity ratios for the c. of g. of the upper rod $AB$, are,

$$K_{x\theta} = -s \sin\theta \quad \text{and} \quad K_{y\theta} = s \cos\theta.$$

The partial velocity ratios can be directly obtained from a velocity resolution. First holding $\phi$ constant, i.e. with respect to the vertical, the motion of $C$ corresponds to that of $B$ since the lower rod can only be subjected to a pure translation, with no angular motion. Then,

$$\dot{x}_c = \dot{x}_b = -l\dot{\theta} \sin\theta, \qquad \dot{y}_c = \dot{y}_b = l\dot{\theta} \cos\theta$$

so that,

$$K_{xc\theta} = K_{xb\theta} = -l \sin\theta, \qquad K_{yc\theta} = K_{yb\theta} = l \cos\theta.$$

If now we hold $\theta$ constant, the partial motion of $BD$ consists of a simple rotation about $B$, so that

$$K_{xc\phi} = -s_1 \sin\phi \quad \text{and} \quad K_{yc\phi} = s_1 \cos\phi.$$

The partial derivatives of the partial velocity ratios, are,

$$\frac{\partial K_{xc\theta}}{\partial\theta} = -l \cos\theta, \qquad \frac{\partial K_{yc\theta}}{\partial\theta} = -l \sin\theta,$$

$$\frac{\partial K_{xc\phi}}{\partial\phi} = -s_1 \cos\phi, \qquad \frac{\partial K_{yc\phi}}{\partial\phi} = -s_1 \sin\phi,$$

and

$$\frac{\partial K_{xc\theta}}{\partial\phi} = \frac{\partial K_{yc\theta}}{\partial\phi} = \frac{\partial K_{xc\phi}}{\partial\theta} = \frac{\partial K_{yc\phi}}{\partial\phi} = 0.$$

The virtual work of the applied forces are,

$$mg\delta x_s + m'g\delta x_c = mgK_{x\theta}\delta\theta + m'g(K_{xc\theta}\delta\theta + K_{xc\phi}\delta\phi),$$

so that the applied forces corresponding to the coördinates $\theta$ and $\phi$ are,

$$mgK_{x\theta} + m'gK_{xc\theta} \quad \text{and} \quad m'gK_{xc\phi}.$$

The equations of motion are respectively,

$$I\ddot{\theta} + m'\left( K_{xc\theta}{}^2\ddot{\theta} + K_{xc\theta}K_{xc\phi}\ddot{\phi} + K_{xc\theta}\frac{\partial K_{xc\theta}}{\partial\theta}\dot{\theta}^2 + K_{xc\theta}\frac{\partial K_{xc\phi}}{\partial\phi}\dot{\phi}^2 + 2K_{xc\theta}\frac{\partial K_{xc\theta}}{\partial\phi}\dot{\theta}\dot{\phi}\right) + m'\left( K_{yc\theta}{}^2\ddot{\theta} + K_{yc\theta}K_{yc\phi}\ddot{\phi} + K_{yc\theta}\frac{\partial K_{yc\theta}}{\partial\theta}\cdot\dot{\theta}^2 + K_{yc\theta}\frac{\partial K_{yc\phi}}{\partial\phi}\dot{\phi}^2 + 2K_{yc\theta}\frac{\partial K_{yc\theta}}{\partial\phi}\dot{\theta}\dot{\phi}\right) = mgK_{x\theta} + m'gK_{xc\theta},$$

$$I_g'\ddot{\phi} + m'\left( K_{xc\phi}{}^2\ddot{\phi} + K_{xc\phi}K_{xc\theta}\ddot{\theta} + K_{xc\phi}\frac{\partial K_{xc\phi}}{\partial\phi}\cdot\dot{\phi}^2 + K_{xc\phi}\frac{\partial K_{xc\theta}}{\partial\theta}\dot{\theta}^2 + 2K_{xc\phi}\frac{\partial K_{xc\phi}}{\partial\theta}\dot{\theta}\dot{\phi} + m'\left( K_{yc\phi}{}^2\ddot{\phi} + K_{yc\phi}K_{yc\theta}\ddot{\theta} + K_{yc\phi}\frac{\partial K_{yc\phi}}{\partial\phi}\cdot\dot{\phi}^2 + K_{yc\phi}\frac{\partial K_{yc\theta}}{\partial\theta}\dot{\theta}^2 + 2K_{yc\phi}\frac{\partial K_{yc\phi}}{\partial\theta}\dot{\theta}\dot{\phi}\right) = m'gK_{xc\phi}.$$

Substituting for the partial velocity ratios and their derivatives, we obtain,

$$(I + m'l^2)\ddot{\theta} + m'ls_1(\cos(\phi - \theta)\ddot{\phi} - \sin(\phi - \theta)\dot{\phi}^2) = -(ms + m'l)g\sin\theta,$$

$$(I_g' + m's_1{}^2)\ddot{\phi} + m'ls_1(\cos(\phi - \theta)\ddot{\theta} + \sin(\phi - \theta)\dot{\theta}^2) = -mgs_1\sin\phi,$$

which are the equations of motion of the system.

We will now consider a different system of coördinates defining the configuration. Let $\theta$ be the angular coördinate of the upper rod as before, but for the lower rod we will consider the relative coördinate $\psi$, that is the angle of the lower rod with respect to the upper rod, Fig. (55*b*).

The coördinates of $l$ for the lower rod $BD$ are

$$x_c = l\cos\theta + s_1\cos(\psi + \theta),$$
$$y_c = l\sin\theta + s_1\sin(\psi + \theta),$$

so that the partial velocity ratios are,

$$K_{x\theta} = -l \sin\theta - s_1 \sin(\psi + \theta), \quad K_{x\psi} = -s_1 \sin(\psi + \theta),$$
$$K_{y\theta} = l \cos\theta + s_1 \cos(\psi + \theta), \quad K_{y\psi} = s_1 \cos(\psi + \theta),$$

which can be obtained directly by a velocity resolution first holding $\psi$ constant and then $\theta$ constant, respectively.

The partial derivatives of the velocity ratios are

$$\frac{\partial K_{x\theta}}{\partial\theta} = -l\cos\theta - s_1\cos(\psi+\theta), \quad \frac{\partial K_{x\theta}}{\partial\psi} = -s_1\cos(\psi+\theta),$$
$$\frac{\partial K_{y\theta}}{\partial\theta} = -l\sin\theta - s_1\sin(\psi+\theta), \quad \frac{\partial K_{y\theta}}{\partial\psi} = -s_1\sin(\psi+\theta),$$
$$\frac{\partial K_{x\psi}}{\partial\psi} = -s_1\cos(\psi+\theta), \quad \frac{\partial K_{x\psi}}{\partial\theta} = -s_1\cos(\psi+\theta),$$
$$\frac{\partial K_{y\psi}}{\partial\psi} = -s_1\sin(\psi+\theta), \quad \frac{\partial K_{y\psi}}{\partial\theta} = -s_1\sin(\psi+\theta),$$

where we note, the reciprocal relation,

$$\frac{\partial K_{x\theta}}{\partial\psi} = \frac{\partial K_{x\psi}}{\partial\theta}, \quad \text{and} \quad \frac{\partial K_{y\theta}}{\partial\psi} = \frac{\partial K_{y\psi}}{\partial\theta}.$$

The total angular displacement of the lower rod $BD$ is $\phi = \theta + \psi$, so that the partial angular velocity ratios are

$$K_{\phi\theta} = \frac{\partial\phi}{\partial\theta} = 1 \quad \text{and} \quad K_{\phi\psi} = \frac{\partial\phi}{\partial\psi} = 1$$

and

$$\frac{\partial K_{\phi\theta}}{\partial\theta} = \frac{\partial K_{\phi\psi}}{\partial\psi} = \frac{\partial K_{\phi\theta}}{\partial\psi} = \frac{\partial K_{\phi\psi}}{\partial\theta} = 0.$$

The virtual work of the applied forces, are,

$$mg\delta x_s + m'g\delta x_c = mgK_{x\theta}\delta\theta + m'g(K_{x\theta}\delta\theta + K_{x\psi}\delta\psi),$$

so that the applied forces corresponding to the coördinates $\theta$ and $\psi$ are,

$$mgK_{x\theta} + m'gK_{x\theta} \quad \text{and} \quad m'gK_{x\psi}.$$

The equations of motion are, respectively,

$$I\ddot{\theta} + I_g'(K_{\phi\theta}{}^2\ddot{\theta} + K_{\phi\theta}K_{\phi\psi}\ddot{\psi}) + m'\Big(K_{xc\theta}{}^2\ddot{\theta} + K_{xc\theta}K_{xc\psi}\ddot{\psi} + K_{xc\theta}\frac{\partial K_{xc\theta}}{\partial\theta}\cdot\theta^2 + K_{xc\theta}\frac{\partial K_{xc\psi}}{\partial\psi}\psi^2 + 2K_{xc\theta}\frac{\partial K_{xc\theta}}{\partial\psi}\theta\psi\Big) + m'\Big(K_{yc\theta}{}^2\ddot{\theta} + K_{yc\theta}K_{yc\psi}\ddot{\psi} + K_{yc\theta}\frac{\partial K_{yc\theta}}{\partial\theta}\theta^2 + K_{yc\theta}\frac{\partial K_{yc\psi}}{\partial\psi}\psi^2 + 2K_{yc\theta}\frac{\partial K_{yc\theta}}{\partial\psi}\theta\psi\Big) = mgK_{x\theta} + m'gK_{xc\theta}$$

and

$$I_g'(K_{\phi\psi}{}^2\ddot{\psi} + K_{\phi\psi}K_{\phi\theta}\ddot{\theta}) + m'\Big(K_{xc\psi}{}^2\ddot{\psi} + K_{xc\psi}K_{xc\theta}\ddot{\theta} + K_{xc\psi}\frac{\partial K_{xc\psi}}{\partial\psi}\cdot\psi^2 + K_{xc\psi}\frac{\partial K_{xc\theta}}{\partial\theta}\cdot\theta^2 + 2K_{xc\psi}\frac{\partial K_{xc\psi}}{\partial\theta}\theta\psi\Big) + m'\Big(K_{yc\psi}{}^2\ddot{\psi} + K_{yc\psi}K_{yc\theta}\ddot{\theta} + K_{yc\psi}\frac{\partial K_{yc\psi}}{\partial\psi}\cdot\psi^2 + K_{yc\psi}\frac{\partial K_{yc\theta}}{\partial\theta}\cdot\theta^2 + 2K_{yc\psi}\frac{\partial K_{yc\psi}}{\partial\theta}\theta\psi\Big) = m'gK_{xc\psi}.$$

Substituting the partial velocity ratios and their derivatives, we obtain

$$I\ddot{\theta} + m'[(l^2 + 2ls_1\cos\psi + s_1{}^2 + k_1{}^2)\ddot{\theta} - 2ls_1\sin\psi\cdot\psi\dot{\theta} + (ls_1\cos\psi + s_1{}^2 + k_1{}^2)\ddot{\psi} - ls_1\sin\psi\dot{\psi}^2] = -(ms + m'l)g\sin\theta - m's_1g\sin(\psi + \theta)$$

and

$$m'[(s_1{}^2 + k_1{}^2)\ddot{\psi} + (ls_1\cos\psi + s_1{}^2 + k_1{}^2)\ddot{\theta} + ls_1\sin\psi\cdot\dot{\theta}^2] = -m'gs_1\sin(\psi + \theta),$$

where $I_g' = m'k_1{}^2$.

### THE REACTIONS AND CHANGE OF ANGULAR MOMENTUM ABOUT FIXED AXIS—DOUBLE PENDULUM.

As an example of the equations of momentum in terms of the velocity ratios, we will now consider the equations for the reaction at the axis, and the rate of change of the angular

momentum about this axis. We will consider the coördinates as $\theta$ and the relative coördinate $\psi$, Fig. (54$b$).

For the components of the reaction at the axis, we have,

$$X = m\left(K_{x\theta}\ddot{\theta} + \frac{\partial K_{x\theta}}{\partial\theta}\cdot\dot{\theta}^2\right) + m'\left(K_{x\theta}\ddot{\theta} + K_{x\psi}\ddot{\psi} + \frac{\partial K_{x\theta}}{\partial\theta}\dot{\theta}^2 + \frac{\partial K_{x\psi}}{\partial\psi}\dot{\psi}^2 + 2\frac{\partial K_{x\psi}}{\partial\theta}\dot{\theta}\dot{\psi}\right),$$

$$Y = m\left(K_{y\theta}\ddot{\theta} + \frac{\partial K_{y\theta}}{\partial\theta}\dot{\theta}^2\right) + m'\left(K_{y\theta}\ddot{\theta} + K_{y\psi}\ddot{\psi} + \frac{\partial K_{y\theta}}{\partial\theta}\cdot\dot{\theta}^2 + \frac{\partial K_{y\psi}}{\partial\psi}\cdot\dot{\psi}^2 + 2\frac{\partial K_{y\psi}}{\partial\theta}\cdot\dot{\theta}\dot{\psi}\right) + (m+m')g.$$

Substituting for the velocity ratios, we have,

$$X = -\left[(ms+m'l)\sin\theta + m's_1\sin(\psi+\theta)\right]\ddot{\theta} - m's_1\sin(\psi+\theta)\ddot{\psi} - \left[(ms+m'l)\cos\theta + m's_1\cos(\psi+\theta)\right]\dot{\theta}^2 - m's_1\cos(\psi+\theta)\dot{\psi}^2 - 2m's_1\cos(\psi+\theta)\dot{\theta}\dot{\psi},$$

$$Y = \left[(ms+m'l)\cos\theta + m's_1\cos(\psi+\theta)\right]\ddot{\theta} + m's_1\cos(\psi+\theta)\ddot{\psi} - \left[(ms+m'l)\sin\theta + m's_1\sin(\psi+\theta)\right]\dot{\theta}^2 - m's_1\sin(\psi+\theta)\dot{\psi}^2 - 2m's_1\sin(\psi+\theta)\dot{\theta}\dot{\psi} + (m+m')g.$$

For the rate of change of angular momentum about the fixed axis,

$$I\ddot{\theta} + I_g'\left(K_{\phi\theta}\ddot{\theta} + K_{\phi\psi}\ddot{\psi} + \frac{\partial K_{\phi\theta}}{\partial\theta}\dot{\theta}^2 + \frac{\partial K_{\phi\psi}}{\partial\psi}\dot{\psi}^2 + 2\frac{\partial K_{\phi\psi}}{\partial\theta}\dot{\theta}\dot{\psi}\right) + m'\left[\left(K_{y\theta}\ddot{\theta} + K_{y\psi}\ddot{\psi} + \frac{\partial K_{y\theta}}{\partial\theta}\dot{\theta}^2 + \frac{\partial K_{y\psi}}{\partial\psi}\dot{\psi}^2 + 2\frac{\partial K_{y\psi}}{\partial\theta}\dot{\theta}\dot{\psi}\right)x_c - \left(K_{x\theta}\ddot{\theta} + K_{x\psi}\ddot{\psi} + \frac{\partial K_{x\theta}}{\partial\theta}\dot{\theta}^2 + \frac{\partial K_{x\psi}}{\partial\psi}\dot{\psi}^2 + 2\frac{\partial K_{x\psi}}{\partial\theta}\dot{\theta}\dot{\psi}\right)y_c\right] = -mgs\sin\theta - m'g(l\sin\theta + s_1\sin(\theta+\psi)),$$

where

$$x_c = l\cos\theta + s_1\cos(\theta+\psi), \quad y_c = l\sin\theta + s_1\sin(\psi+\theta).$$

Substituting for the velocity ratios and noting $I_g' = m'k_1^2$ we obtain

$$I\ddot{\theta} + m'[(l^2 + 2ls_1 \cos \psi + s_1^2 + k_1^2)\ddot{\theta} + (ls_1 \cos \psi + s_1^2 + k_1^2)\ddot{\psi} - ls_1 \sin \psi \cdot \dot{\psi}^2 - 2ls_1 \sin \psi\dot{\psi}\dot{\theta}] = -mgs \sin \theta - m'g(l \sin \theta + s_1 \sin (\theta + \psi)),$$

which agrees with the generalized equation for the coördinate $\theta$ obtained previously.

OSCILLATION OF ROLLING CYLINDERS FIG. 55.

We will now consider the oscillation of a system consisting of two cylinders, an inner cylinder which rolls along the inside circumference of an outer cylinder which latter is free to rotate about a fixed axis at its center, Fig. (55).

Let $b$ = radius of outer cylinder
$a$ = radius of inner cylinder
$I$ = moment of inertia of outer cylinder
$M$ and $k$ = mass and radius of gyration of the inner cylinder
$\theta$ = angle of the radius vector of the line joining the centers of outer and inner cylinder centers, with respect to the vertical
$\phi$ = angle turned by the outer cylinder
$\psi$ = angle turned by the inner cylinder

Then

Holding $\phi$ constant, $(b - a)\theta - \psi a = 0$,

$$\therefore \frac{\psi}{\theta} = K_{\psi\theta} = \frac{b - a}{a},$$

Holding $\theta$ constant, $b\phi = -a\psi$,

$$\frac{\psi}{\phi} = K_{\psi\phi} = -\frac{b}{a},$$

and we note,

$$\frac{\partial K_{\psi\theta}}{\partial \theta} = \frac{\partial K_{\psi\phi}}{\partial \phi}, \qquad \text{etc.} = 0.$$

Further if $s$ is the displacement of the center of inner cylinder, we have

$$\dot{s} = (b - a)\dot{\theta}, \qquad \therefore \quad K_{s\theta} = b - a.$$

The vertical displacement $y$ of the axis of the inner cylinder is

$$y = (b - a)\cos\theta, \qquad K_{y\theta} = -(b - a)\sin\theta.$$

For the coördinate $\theta$, the equation of motion is

$$Mk^2(K_{\psi\theta}{}^2\ddot{\theta} + K_{\psi\theta}K_{\psi\phi}\ddot{\phi}) + MK_{s\theta}{}^2\ddot{\theta} = MgK_{y\theta}$$

and for the coördinate $\phi$, the equation of motion is

$$I\ddot{\phi} + Mk^2(K_{\psi\phi}{}^2\ddot{\phi} + K_{\psi\phi}K_{\psi\theta}\ddot{\theta}) = 0.$$

Substituting for the velocity ratios, we obtain

$$\left(Mk^2\left(\frac{b-a}{a}\right)^2\ddot{\theta} - \frac{b(b-a)}{a^2}\ddot{\phi}\right) + M(b-a)^2\ddot{\theta} = -Mg(b-a)\sin\theta$$

or

$$\left(1 + \frac{k^2}{a^2}\right)(b-a)\ddot{\theta} - \frac{bk^2}{a^2}\ddot{\phi} = -g\sin\theta$$

and

$$I\ddot{\phi} + Mk^2\left(\frac{b^2}{a^2}\ddot{\phi} - \frac{b}{a^2}(b-a)\ddot{\theta}\right) = 0.$$

Eliminating $\ddot{\phi}$, the equation of oscillation is

$$(b-a)\left(1 + \frac{Ik^2}{Ia^2 + Mk^2b^2}\right)\ddot{\theta} + g\sin\theta = 0.$$

INTERNAL REACTIONS, FIG. 55.

The condition of rolling requires a constraint reaction in the nature of a tangential frictional component between the two rolling disks. The system may be regarded as having three degrees of freedom provided we include the condition of constraint due to rolling.

The degrees of freedom are $\theta$, $\psi$ and $\phi$ with the condition of constraint

$$(b-a)\dot{\theta} - \dot{\psi}a = b\dot{\phi}$$

or

$$(b-a)\delta\theta - a\delta\psi - b\delta\phi = 0.$$

The kinetic energy of the system is

$$T = \tfrac{1}{2}(I\dot{\phi}^2 + M(b-a)^2\dot{\theta}^2 + Mk^2\dot{\psi}^2)$$

and the equations of motion are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) - \frac{\partial T}{\partial \phi} = -\lambda b,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = \lambda(b-a) - Mg(b-a)\sin\theta,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right) - \frac{\partial T}{\partial \psi} = -\lambda a;$$

hence

$$I\ddot{\phi} = -\lambda b, \qquad M(b-a)\ddot{\theta} = \lambda - Mg\sin\theta$$

and

$$Mk^2\ddot{\psi} = -\lambda a.$$

Evidently, the multiplier $\lambda$ is numerically equal to the frictional component between the two cylinders.

The normal component between the cylinders is

$$N = Mg\cos\theta + M(b-a)\dot{\theta}^2$$

and the horizontal and vertical components at the fixed axis of the outer cylinder are

$$\lambda\cos\theta - N\sin\theta \quad \text{and} \quad \lambda\sin\theta + N\cos\theta + W,$$

where

$$\dot{\theta} = \sqrt{\frac{2g}{A}(\cos\theta - \cos\theta_0)},$$

$$A = (b-a)\left(1 + \frac{Ik^2}{Ia^2 + Mk^2b^2}\right),$$

$W$ = weight of outer cylinder.

*(To be continued)*

**Changes in the Dimensions of Soft Drawn Copper Wire Produced by Torsion.** THOMAS LONSDALE. (*Phil. Mag.*, Nov., 1929.) Ercolini in 1906 discovered that torsion applied cyclically to a loaded copper wire made it lengthen. A few years later Poynting proved mathematically that a cylinder of a perfectly elastic solid on which a pure torsional shear acts should show a small elongation proportional to the square of the twist. This prediction he verified by experiments on iron and steel wires in which the torsion did not exceed the elastic limit. In the experiments about to be described the conduct of high conductivity soft drawn copper wire at room temperature was examined when tension and torsion were simultaneously applied. The tensions were too small to cause permanent elongation when they acted by themselves. The torsions far exceeded the elastic limits and were increased until the wire broke. The large elongations recorded manifestly belong to the copper after its elastic limit had been passed under the combined action of tension and torsion.

The length of wire on which the changes of length were observed was usually 175 cm. The clamp holding the lower end of the wire in a vertical position had no freedom of motion up and down but could be turned around to give torsion to the wire. The upper end also was supported in a clamp. This was kept free from rotation and supported by a thread passing over a pulley. From the free end of the thread hung weights to counterbalance the clamps and wire and in addition to apply tension to the wire. Elongations of the wire were observed by reading the upward motion of the upper clamp. The tensions applied ranged from 9 to 515 kg./sq. cm. Even the largest of these tensions acting for 24 hours produced no permanent lengthening of the wire to which it was applied.

Different loads were applied to the wire and for each load the wire was twisted through varying numbers of turns per cm. of length. For copper wire of diameter .012 cm. this number was as great as 60. This wire twisted to the extent just stated and carrying a load of 14 grams in addition to an initial tension amounting to 124 km./sq. cm. was elongated by 5.97 per cent. of its initial length. The greatest elongation was reached with copper wire .0715 cm. in diameter loaded with 510 grams in addition to an initial tension of 127 kg. per sq. cm. When it was twisted 14 times per cm., it was elongated 9.13 per cent. of the initial length.

"The elongation for a given twist and a given tension is roughly proportional to the diameter of the wire. The mean percentage elongations at which break takes place for the four sets of curves, namely 7.7, 6.4, 5.5, 4.2, show that the thinner wires tend to break at a less extension than the thicker ones, but that the tendency to break is by no means proportional to the diameter of the wire."

G. F. S.

## NOTES FROM U. S. BUREAU OF STANDARDS.*

### NEW FUNDAMENTAL FREQUENCY STANDARD.

The bureau has recently installed equipment which materially advances the accuracy of the frequency standard. The new equipment comprises essentially a group of four piezo oscillators, each having the frequency 100 kilocycles. Three are alternative standards, the fourth is a reference point against which to check the others. Beats between each of the first three and the fourth are automatically counted by three telephone message registers. An automatic camera takes a picture of the counters each 1,000 seconds, from which record the number of beats per 1,000 seconds of each standard against the reference point can be obtained. In 1,000 seconds, each oscillator makes 100 million oscillations, so that variations in frequency within the group can be measured to one part in one hundred million.

To obtain the absolute frequency, the output of one oscillator is fed into a submultiple generator from which currents of 10 kilocycles and 1 kilocycle may be drawn, these frequencies being as accurate as the original oscillations. The 1-kc. frequency drives a synchronous motor clock, which is geared to keep exact mean solar time when the input frequency is exactly 1,000 cycles.

The rate of the clock is obtained by checks with Arlington time signals. The percentage gain or loss then is numerically equal to the deviation of the oscillator from 100 kc. That is, if the clock gained one second per day, it would be fast one part in 86,400, so the frequency of the piezo oscillator is 100,001.16 cycles per second.

The crystals are of 30° cut, and vibrate on a thickness frequency. They are doughnut-shaped, that shape being chosen as giving a low temperature coefficient. The temperature and atmospheric pressure in which the crystals operate are carefully regulated, as are the filament and plate voltages. Standby batteries take care of power failures.

* Communicated by the Director.

Measurements show that the average short time variations of each of the crystals are less than one part in ten million. The standard maintains an absolute value of frequency which is known to one part in ten million.

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### AERONAUTIC RADIO BEACON IMPROVEMENTS.

Recent work at the bureau has led to improvements in the characteristics of the vibrating reed course indicator, resulting in a 20 per cent. increase in sensitivity and at the same time reducing the required constancy to which the modulation frequencies at the beacon station must be held. A variation of $\pm$ 0.75 per cent. in these frequencies is now permissible. These improvements have been obtained through the use for the reed material of a high-permeability nickel-steel alloy, called "A" metal, rather than elinvar which was previously employed.

A small filter unit was designed permitting the simultaneous connection of the reed indicator and the head telephones in the receiving set output without materially affecting the performance of the reed indicator. This unit operates to reduce to a negligible degree the low-frequency hum heard in the ear phones when visual type beacon signals are received. It therefore permits the pilot to wear the head-telephones while the reeds are in operation, if desired, so that when the visual range beacon is interrupted for transmitting a radio telephone broadcast the pilot will hear the broadcast begin.

A receiving set installation was made to permit night flights on the College Park beacon. On one night flight, made from College Park, Md. to Philadelphia, Pa., and return, no course variations were observed. The distance from College Park to Philadelphia is 115 miles. In a number of previous tests made on the ground at Media, Pa., 105 miles from College Park, some perceptible night course variations had been obtained. At a distance of 200 miles from College Park, on the ground at Boonton, N. J., large variations have been observed, which might seriously limit the usefulness of the beacon at night at this distance. The result of the night test flight to Philadelphia appeared to show that variations in the air are less than ground tests indicate. A series of night flights are planned to secure more data on dependable distance ranges at night.

## MEETING OF AMERICAN PHYSICAL SOCIETY.

The 163d regular meeting of the American Physical Society was held in Washington April 24–26, inclusive. Sessions of the first two days were held at the Bureau of Standards and on the last day at the National Academy of Sciences.

Five papers were presented by members of the bureau's staff, as follows:

(1) The mechanism of atomization, by R. A. Castleman; a new theory which appears to explain the process of liquid "atomization" is satisfactorily set forth.

(2) Wind pressure on cylindrical stacks, by H. L. Dryden and G. C. Hill, in which a summary was presented of published model experiments on the wind pressure on cylinders and of some additional experiments on model cylinders in the wind tunnel, on a large cylinder in natural wind, and on the stack of the bureau's new power plant in natural wind.

(3) Airfoils of circular-arc section for use at high speeds, by L. J. Briggs and H. L. Dryden, in which experiments are described showing that the airfoil sections used in the design of airplane propellers change their aerodynamical properties as the relative airspeed approaches the speed of sound.

(4) Regularities in the spectra of lutecium, by W. F. Meggers and B. F. Scribner, in which the recent analysis of the lutecium spectra and the separation of the lines into three distinct classes are described. (See Technical News Bulletin, No. 156, p. 32, April, 1930.)

(5) Time lag in changes of electrical properties of rubber with temperatures and pressure, by A. Scott, in which it is shown that when the temperature of rubber is suddenly changed, the electrical properties do not at once assume their final values. Two hours or more may be required for the dielectric constant, power factor, and resistivity to become constant.

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## MEETING OF SEISMOLOGICAL SOCIETY OF AMERICA.

On May 5 and 6 the eastern section of the Seismological Society of America held its annual meeting in Washington as a joint meeting with the seismology section of the American Geophysical Union. The meeting on May 5 was held at the Bureau of Standards and on May 6 at Georgetown University.

In addition to the usual business meeting, the session at the bureau on the morning of May 5 included an address of welcome by Dr. George K. Burgess; a paper on engineering seismology in Japan by Charles W. Brown, with notes and comments by R. R. Martel; a discussion of the need of additional seismological data by structural engineers, by John R. Freeman; and a paper on the relation between the engineering and the seismological attack on the earthquake problem, by N. H. Heck.

Luncheon was served at the bureau and in the afternoon seven additional papers were presented, including one on the coördination of seismological investigation in the United States by Dr. R. S. Patton, Director of the United States Coast and Geodetic Survey.

The session at Georgetown University included a business meeting, election of officers and the presentation of additional papers. Those attending the meeting inspected the Georgetown Seismograph Station, where the late Father Tondorf did so much valuable work.

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### DETERMINATION OF OSMIUM.

Further work indicates that the incomplete recovery of osmium, referred to in earlier reports on the development of a method for the determination of osmium (Technical News Bulletin No. 156, p. 32; April, 1930), is caused by the presence of a small amount of sulphuric acid formed by the oxidation of the sulphur dioxide which is used to reduce the osmium tetroxide. When the absorbing solution is evaporated to dryness to expel hydrochloric acid and sulphur dioxide, and to convert complex sulphites of osmium to the chloride, the presence of the sulphuric acid appears to cause a slight volatilization of osmium tetroxide.

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### PURE IRIDIUM.

Two lots of iridium which are being purified were subjected to several fractional precipitations with hydrogen sulphide. Small samples were taken at several stages and subjected to

spectrographic examination. The results indicate that the concentrations of all platinum metal impurities decrease with repeated treatments with hydrogen sulphide. In one of the iridium solutions platinum seems to have been practically eliminated, and ruthenium is present in very small amounts. The other solution is still somewhat less pure but the elimination of platinum and ruthenium seems to be progressing. Palladium has been eliminated from both solutions. Rhodium is the most difficult of the platinum metals to eliminate by this method.

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## EFFECT OF BORIC ACID ON THE CLINKERING OF PORTLAND CEMENT.

An investigation to determine the possibility of producing a well burned clinker at a relatively low temperature, by the use of a small quantity of boric acid as a flux, was begun several months ago. The energy changes which take place during burning were studied by means of heating curves. A study of these curves led to a brief investigation of the various constituents normally present in Portland cement.

Heating curves of mixtures of $CaCO_3$ and $SiO_2$ in the ratio of $2CaO : SiO_2$ gave evidence of (1) an exothermic reaction at about 1420° C., and (2) an endothermic reaction immediately following, and probably starting before the first reaction is finished. The first is thought to be the formation of $2CaO.SiO_2$, as no indication of it is found on the cooling curve, or upon reheating. Heating curves of $2CaO.SiO_2$ clinker fail to show this break.

The second reaction is reversible, and is probably the inversion of beta $2CaO.SiO_2$ to the alpha form, as the temperature at which it occurs corresponds with the generally accepted temperature for this transition.

The first reaction is not affected by the addition of small quantities of boric acid to the raw mixture, but the second was found to occur at progressively lower temperatures with increasing amounts of boric acid. The refractive indices of the clinkers were also lowered by the addition of boric acid, and from these two facts it is held possible that some of the boric acid forms a solid solution with the silicate.

Mixtures of CaO, $SiO_2$, $Al_2O_3$, $Fe_2O_3$, and MgO approximating the composition of Portland cement, gave curves similar to those for the CaO–$SiO_2$ mixtures. The exothermic break came at about 1300° C. in these mixtures. The addition of boric acid had little apparent effect on the shape of the curves. The clinkers containing boric acid, however, were better burned than those without boric acid. Contrary to expectation, the clinkers containing boric acid also had a higher percentage of free lime than the others.

It is planned to make a systematic study of the portion of the ternary system CaO–$SiO_2$–$B_2O_3$ in the region between $CaO.SiO_2$, $3CaO.SiO_2$, and 5 per cent. $B_2O_3$, in order to interpret the phenomena already observed as well as to obtain additional information. Other systems may be studied later, if it appears advisable.

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## HEAVY CLAY INVESTIGATION.

This investigation is being conducted at the Columbus branch of the bureau and has for its object the determination of the properties which characterize a number of representative Ohio State clays used primarily in the manufacture of so-called heavy clay ware, which includes brick, hollow tile, paving block, etc. The first news item on the investigation was published in Technical News Bulletin No. 154 (February, 1930) and presents the mineralogical composition of 27 clays. The present report presents values obtained in determining the softening points of the clays. The values are given in terms of pyrometric cones (P. C. E. or pyrometric cone equivalent value). The values found are as follows:

| Type of Material. | P. C. E. Value. |
|---|---|
| Shales: | |
| Allegheny, Malvern, Ohio | 9 |
| Allegheny, New Lexington, Ohio | 8 |
| Bedford, Columbus, Ohio | 7–8 |
| Chagrin, Cleveland, Ohio | 5 |
| Cincinnati, Cincinnati, Ohio | 1–2 |
| Connemaugh, Ava, Ohio | 8–9 |
| Connemaugh, Summitville, Ohio | 8–9 |
| Dunkard, Marietta, Ohio | 4–5 |
| Pottsville, Portsmouth, Ohio | 13 |
| Waverly, Waverly, Ohio | 10–11 |

| | |
|---|---|
| Glacial: | |
| Findlay | 5–6 |
| Lima | 1 |
| London | 5–6 |
| Mt. Healthy | 11 |
| New London | 5–6 |
| Toledo | 03–02 |
| Upper Sandusky | 2–3 |
| Alluvial: | |
| Beaver | 9 |
| Gallipolis | 10 |
| Mowrystown | 12 |
| Rutland | 4–5 |
| Sugar Creek | 8 |
| Westerville | 11 |
| Zanesville | 6 |
| Zanesville (loess) | 12 |

## SPECIFICATIONS FOR ROOFING SLATE.

A few months ago this bureau was called upon by the Federal Specification Board for information on roofing slate to form the basis of a specification for this material. After a study of the data available it was concluded that the information on weathering qualities was too meager, and the significance of various tests in this connection was not thoroughly understood. It was decided, therefore, to make a study of weathered slates which had been in service for a considerable period of time to determine what weathering agents are mostly concerned in the destruction of slate roofs and what qualities in slate are most effective in resisting such destructive agents. Several hundred samples were collected, some of which had been on roofs for more than 100 years. These were carefully studied to determine what alterations had taken place, especial attention being given to the decayed portions.

Slate being a metamorphosed clay sediment, is composed largely of chemically stable elements. It is a very dense material with a higher flexural strength than any other common type of rock. With such properties it might be expected to be almost immune to weathering effects. Nevertheless it is evident that some slates cannot be depended upon for more than 30 years of service on a roof.

Frost action which is commonly assumed to be very destructive to masonry has been found to play a very small part

in the weathering of slate. The decay of slate on a roof is confined mainly to the covered portions of the shingles and manifests itself by a slow process of crumbling and scaling in thin layers. Chemical and petrographic examination of the decayed portions reveal the presence of considerable calcium sulphate, although this compound is almost totally absent in the fresh slate. Some slates contain as much as 4 per cent. of calcium carbonate which could be converted to calcium sulphate by rainwater bearing traces of sulphuric acid. This led to experiments on slate which involved a soaking in very dilute sulphuric acid. Such tests on slates containing appreciable amounts of calcium carbonate caused a surface action similar to that shown on badly weathered slates, but underneath the thin layers so affected the slate was still sound. This seemed to indicate that the action of sulphuric acid is not the only agent concerned in slate weathering.

Evidence of a hydration process has been found, and is being investigated. Soaking and drying tests have shown that some slates can be destroyed in 20 or 30 cycles of such treatment. While it is probably too early to draw conclusions, it seems likely that the hydration effects go hand in hand or, perhaps, precede the decomposition of carbonates.

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## PROPERTIES OF FIBER INSULATING BOARDS.

The bureau has tested various fiber insulating boards, representative of different commercial types as regards both structure and fiber components. The boards were composed variously of fibers from wood, cornstalks, licorice root, and sugar cane (bagasse). The results of the tests are given in the table. Following are the more important details of the tests applied. For further details the report of the bureau's previous study of fiber wall boards may be consulted. (A Study of Fiber Wall Boards for Developing Specification Standards, *Paper Trade Journal*, Vol. 89, No. 13, p. 61; Sept. 16, 1929.)

The behavior of the boards in humid air was determined as follows: Specimens were cut from each material 3 by 12 inches, the longer dimension being crosswise of the board in all cases where crosswise direction of the board was known. The specimens were conditioned for 72 hours at approximately 30

per cent. relative humidity, and then for a like period of time in saturated water vapor. The increase in length, weight, and thickness as a result of absorption of moisture was determined. Each result given is the average of five tests. There was not much variation in the individual tests for a given material.

The flexural strength of the boards was determined by the following method. Specimens 3 by 14 inches were cut from each board, and since the machine and crosswise directions of these boards were not known, 5 samples were cut in each direction and designated as the long and short directions of the board. The strength tests were made on a tensile testing machine equipped with an attachment for determining the flexural strength. The results listed are the loads required to break the boards based on the average of 5 tests in each direction of the board. The deflection was measured at the time of rupture. The individual test results on these boards are very consistent, for the average deviation from the average of the 5 tests is less than 5 per cent.

*Tests of Fiber Insulating Boards.*

| B. S. No. | Density at 30% R. H. g/cm.³ | Weight at 30% R. H. Lbs./-1000 sq. ft. | Thickness at 30% R. H. Inches. | Effect in saturated water vapor. | | | Flexural strength.* | | Deflection at rupture. | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Increase in weight. Per cent. | Expansion. | | Length Pounds. | Width Pounds. | Length Inches. | Width Inches. |
| | | | | | Width of board. Per cent. | Thickness of board. Per cent. | | | | |
| 311001 | 0.34 | 806 | 0.45 | 10.1 | 0.23 | 6.2 | 13.4 | 12.2 | 0.70 | 0.80 |
| 311002 | .26 | 603 | .44 | 13.8 | .26 | 5.0 | 9.8 | 9.5 | .65 | .80 |
| 311003 | .29 | 687 | .45 | 16.3 | .38 | 7.1 | 17.2 | 13.4 | .70 | .80 |
| 311004 | .32 | 745 | .45 | 12.4 | .54 | 5.5 | 13.3 | 9.2 | .70 | .80 |
| 311005 | .22 | 595 | .44 | 14.9 | .47 | 6.4 | 12.8 | 10.0 | .60 | .70 |
| 414031 | .27 | 626 | .44 | 17.6 | .55 | — | 14.6 | 14.2 | .50 | .60 |
| Average | .28 | 677 | .44 | 14.2 | .41 | 6.0 | 13.5 | 11.4 | .64 | .75 |

* Rupturing load at midspan of a specimen 3 inches wide and making a 12-inch span between supports.

**Test for Boron.** J. B. KASEY (*Chemist Analyst*, 1930, *19* No. 2, 15) states that a mixture of sulphuric and hydrofluoric acids is preferable to Turner's flux (potassium hydrogen sulphate and calcium fluoride) in the test for boron, since the purple color of potassium is eliminated. The material to be tested is placed in a clean platinum dish or crucible, 3 or 4 cc. of 48 per cent. hydrofluoric acid and 6 drops of concentrated sulphuric acid are added; the contents of the vessel are heated until an abundant evolution of vapors occurs. The flame is then held over the vessel at a distance of 0.5 to 1.5 inches. Alternate heating and testing with the flame are continued until fumes of sulphur trioxide are evolved. If boron be present, the flame will be colored green by the boron fluoride. Barium does not interfere with the test, but copper must be removed before the test is applied.

J. S. H.

**Cystine Content of Phaseolin.** Phaseolin is a globulin and forms the major portion of the protein of the white, navy, or kidney bean, *Phaseolus vulgaris*. Feeding experiments have indicated that phaseolin is deficient in cystine, which is di alpha amino beta thio propionic acid. M. X. SULLIVAN (*Pub. Health Rep.*, Supplement No. 80, 1929, 7 pages) has made an extensive series of chemical tests on phaseolin, and finds that this globulin contains little if any cystine. The chemical and the biological tests thus agree.

J. S. H.

**Stiffness in Fabrics Produced by Starch.** ESTER C. PETERSON AND TOBIAS DANTZIG (*U. S. Dep. Agric. Tech. Bull.* No. 108, 29 pages, 1929) find that the stiffest fabric is produced by wheat starch, the least stiff by potato starch, while practically the same intermediate effect is produced by corn starch and rice starch. Starch of the same species but from different sources may exhibit different stiffening properties. The stiffness of the sized fabric is not influenced by varying the period of heating of the starch paste within the limits 5 minutes to 60 minutes. A decrease in the concentration of the starch paste tends to produce a gradual decrease in the stiffness of the sized fabrics; this decrease is least marked with potato starch which yields an extremely viscous paste. The stiffening power of corn starch paste is increased by addition of borax in quantities up to 1 part of borax for 5 parts of starch. Addition of salt, paraffin, beeswax, and hydrogenated vegetable oil up to 20 per cent. of the weight of the starch has no appreciable effect upon the stiffness of the fabric sized with corn starch.

J. S. H.

# THE FRANKLIN INSTITUTE.

## STATED MONTHLY MEETING, MAY 21, 1930.

The stated monthly meeting of the Institute was called to order at three-thirty P.M., by the President of the Institute, Mr. Nathan Hayward. The celebration of Medal Day was held in connection with this meeting.

The Chairman stated that the minutes of the April meeting had been printed in full in the May number of the JOURNAL of the Institute and that unless there were objection they would be approved as printed. No objection was offered so the President declared the minutes approved.

The Chairman then called upon the Secretary of the Institute, who reported concerning the affairs of the Institute since May 1929. He referred particularly to the bequest of Dr. William Henry Trueman of $28,300, which had been received during the year. The following portion of the Will was read: "Item: I give and bequeath unto the library of The Franklin Institute of Philadelphia, all my photographic works, periodicals and pamphlets, bound and unbound; also any of my scientific works they may select." Then after reciting various bequests, the Will reads: "All the rest, residue and remainder of my said principal In Trust, I give and bequeath unto The Franklin Institute of Philadelphia, in appreciation of the educational value its lectures and library have been to me from the beginning of my professional career, November 1857 to date" (1912).

The Secretary referred to the helpful magnitude of the bequest and with an especial gratification to the reason which Dr. Trueman gave for making the bequest.

The Secretary further stated that during the year Mr. Samuel Insull of Chicago, founder of the Franklin Medal, had made a handsome addition to that fund in order to give to the Institute the ability to have the Franklin Medalists present at the Medal Day Exercises and to receive their medals in person. He made grateful acknowledgment of the value of this additional benefaction, not only to The Franklin Institute but to the interest of science in general and international good feeling among scientists.

The meeting of the day was made notable by the presence of Mr. Samuel Insull, the founder of the medal, and of the Franklin Medalists of the day, Dr. John F. Stevens and Sir William H. Bragg.

The Secretary made brief references to the progress of the movement for the establishment on the Parkway of the Benjamin Franklin Memorial and The Franklin Institute Museum. He stated that rapid progress was being made and that the outlook for a completely successful issue of the movement was bright. He announced further that it was expected to break ground on June 18, and that the actual work of the erection of the building would be started in the autumn. It is hoped that Medal Day exercises two years hence may be held in the new home of the Institute.

The President then announced that the award of Certificates of Honorary Membership, Certificates of Merit, and Medals, was the next order of business, and stated that all the medals which had been recommended during the year by the Committee on Science and the Arts, and which had been awarded by the Board of Managers, would be presented. The exercises in connection with these awards were then held.

**PROGRAMME.**

| | *Sponsors* |
|---|---|
| Honorary Membership | |
| MORTIMER ELWYN COOLEY, LL.D., ENG.D., Sc.D.<br>Dean Emeritus—College of Engineering<br>University of Michigan | Dr. Howard McClenahan |
| HENRY LEFFMANN, M.D.<br>The Franklin Institute | Dr. Howard McClenahan |
| Presentation of Certificate of Merit to | |
| HEYMAN ROSENBERG<br>Parker-Kalong Corporation<br>New York City | Mr. Henry B. Allen |
| Presentation of Longstreth Medals to | |
| ERVIN GEORGE BAILEY<br>Fuller-Lehigh Company, Fullerton, Pa.<br>Bailey Meter Company, Cleveland, Ohio | Mr. Coleman Sellers, 3d |
| CHARLES WEYL, M.S. IN E.E.<br>Assistant Professor Electrical Engineering<br>University of Pennsylvania | Dr. H. Jermain Creighton |
| Presentation of Wetherill Medals to | |
| CHARLES SUMNER CHRISMAN<br>West Chester, Pennsylvania | Mr. Harold Calvert |
| WILLIAM NICHOLSON JENNINGS<br>Philadelphia, Pennsylvania | Mr. Lionel F. Levy |
| Presentation of the Levy Medal to | |
| FLOYD KARKER RICHTMYER, PH.D.<br>Professor of Physics<br>Cornell University | Dr. Frederic Palmer, Jr. |
| Presentation of Henderson Medals to | |
| GEORGE HANNAUER (posthumous award)<br>Represented by George Hannauer, Jr.<br>Chestnut Hill, Massachusetts | Mr. Walter C. Wagner |
| EDGAR MARVIN WILCOX<br>Hannauer Car Retarder Company<br>Gibson, Indiana | Mr. Walter C. Wagner |
| Presentation of the Clark Medal to | |
| HENRY LATHAM DOHERTY<br>Cities Service Company<br>New York City | Mr. J. M. Rusby |
| Presentation of Cresson Medals to | |
| NORMAN ROTHWELL GIBSON | Mr. Frank H. Rogers |

The Niagara Falls Company
Niagara Falls, New York

IRVING EDWIN MOULTROP — Mr. M. M. Price
The Edison Electric Illuminating Company
Boston, Massachusetts

Presentation of the Franklin Medal and Certificate of Honorary Membership to

JOHN FRANK STEVENS, LL.D. — Dr. James Barnes
Former Chief Engineer—The Panama Canal
Baltimore, Maryland

Presentation of the Franklin Medal and Certificate of Honorary Membership to

SIR WILLIAM HENRY BRAGG, K.B.E., C.B.E., F.R.S., D.C.L., LL.D., D.Sc. — Dr. James Barnes
Director—Royal Institution of Great Britain
London, England

**PAPERS.**

"A Momentous Hour at Panama," by DR. JOHN FRANK STEVENS.
"The Meaning of the Crystal," by SIR WILLIAM HENRY BRAGG.

HOWARD MCCLENAHAN,
*Secretary.*

(An extended account of the proceedings of this Medal Day meeting will appear in the July number of the JOURNAL.)

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## COMMITTEE ON SCIENCE AND THE ARTS.

*(Abstract of Proceedings of Stated Meeting held Wednesday, May 14, 1930.)*

HALL OF THE COMMITTEE,
PHILADELPHIA, May 14, 1930.

MR. HENRY B. ALLEN *in the Chair.*

The following reports were presented for final action:

No. 2908: Scattering and Diffraction of Electrons by Crystals.

This report recommended the award of the Elliott Cresson Medal both to Doctor C. J. Davisson and to Doctor L. H. Germer of the Bell Telephone Laboratories, Inc., New York City,

"In consideration of their pioneer work on the scattering and diffraction of electrons by crystals."

No. 2911: Work of Doctor Kotaro Honda in Magnetism and Metallurgy.

This report recommended the award of the Elliott Cresson Medal to Doctor Kotaro Honda, Director of the Research Institute for Iron, Steel and Other Metals, of the Imperial University of Sendai, Japan,

"In consideration of the valuable contributions that he has made to our knowledge of magnetism, ferro-magnetism and metallurgy."

The following report was presented for first reading:

No. 2920: Enduro K. A. 2 Nirosta Steel.

GEO. A. HOADLEY,
*Secretary to Committee.*

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## LIBRARY NOTES.

### RECENT ADDITIONS.

American Institute of Electrical Engineers. Transactions, January 1930. Volume 49, No. I. 1930.

BELLING, JOHN. The Use of the Microscope: a Handbook for Routine and Research Work. First edition. 1930.

BLOCH, EUGÉNE. L'Ancienne et la Nouvelle Théorie des Quanta. 1930.

BRITTON, HUBERT T. S. Hydrogen Ions. 1929.

CAMERON, JAMES R., AND JOHN F. RIDER. Sound Pictures and Trouble Shooters Manual. 1930.

CAMPBELL, NORMAN ROBERT, AND DOROTHY RITCHIE. Photoelectric Cells: their Properties, Use, and Applications. 1929.

CORSE, W. M. Bearing Metals and Bearings. 1930.

CORDIER, MAXIME. Le Multiplicateur de trois Cents Carrés. 1872.

DAVIES, CECIL W. The Conductivity of Solutions and the Modern Dissociation Theory. 1930.

DRESDEN, ARNOLD. Solid Geometry and Determinants. 1930.

DURRANS, THOS. H. Solvents. 1930.

EDWARDS, JUNIUS DAVID, FRANCIS C. FRARY AND ZAY JEFFRIES. The Aluminum Industry. Two volumes. 1930.

FLEMING, ROBINS. Wind Stresses in Buildings. 1930.

HUMPHREY, D. Advanced Mathematics for Students of Physics and Engineering. Two parts in one volume. 1929.

KNIGHT, AUSTIN M. Modern Seamanship. Ninth edition, rewritten, revised and enlarged by Harry A. Baldridge. 1930.

MALLETT, E. Telegraphy Including Wireless. 1929.

MOLDENKE, RICHARD. The Principles of Iron Founding. Second edition. 1930.

NASH, ALFRED W., AND A. R. BOWEN. The Principles and Practice of Lubrication. 1929.

National Advisory Committee for Aeronautics. Bibliography of Aeronautics, 1928.

National Research Council. Annual Survey of American Chemistry. Volume IV, July 1, 1928 to December 31, 1929. 1930.

PLINY (CAIUS PLINIUS SECUNDUS). The Elder Pliny's Chapters on Chemical Subjects. Part I, edited, with translations and notes, by Kenneth C. Bailey. 1929.

ROBINSON, CLARK SHOVE. The Elements of Fractional Distillation. Second edition, 1930.

ROLT, F. H. Gauges and Fine Measurements. Two volumes. 1929.
SHARP, M. A., AND W. M. SHARP. Principles of Farm Mechanics. 1930.
TRUMPER, MAX. Memoranda of Toxicology. Partly based on Tanners Memoranda of Poisons. Second edition. 1929.
WHETHAM, WILLIAM CECIL DAMPIER DAMPIER. A History of Science and Its Relations with Philosophy and Religion. 1929.
WIEN, W., AND F. HARMS, editors. Handbuch der Experimentalphysik. Band 10. 1930.

## BOOK REVIEWS.

HANDBOOK OF CHEMISTRY AND PHYSICS. By Charles D. Hodgman, M.S. and Norbert A. Lange, Ph.D. Fourteenth Edition. 1386 pages, 12mo, limp leather. Cleveland, Chemical Rubber Publishing Company, 1929. Price, $5.00.

Books of this character, now fairly numerous and applicable to almost all the applied sciences, are of great use to those who are in regular scientific work, containing as they do a vast amount of data not readily available elsewhere. The merit of a book of this kind will depend upon the judgment in selecting the data, upon the care with which the figures are transcribed, the mechanical execution, quality of the paper, clearness of type and convenience in size.

The book in hand having reached a fourteenth edition has shown its merits. It corresponds to the requirements just stated, and is a very valuable addition to the library of the chemist or physicist.

NATIONAL ADVISORY COMMITTEE FOR AËRONAUTICS.

Report No. 330. Experimental and Analytical Determination of the Motion of Hydraulically Operated Valve Stems in Oil Engine Injection Systems. By A. G. Gelalles and A. M. Rothrock. 20 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

This research on the pressure variations in the injection system of the N.A.C.A. spray photography equipment and on the effects of these variations on the motion of the timing valve stem was undertaken in connection with the study of fuel injection systems for high-speed oil engines. The spray photography equipment employed for these tests consists of a fuel injection system for producing an oil spray, an electrical spark system for illuminating the spray, and a photographic camera for recording its development. The fuel injection system contains a high-pressure hand pump for producing the injection pressures, an oil reservoir for maintaining the pressures of the fuel during the injection, a timing valve for timing the start of the oil spray, an injection valve for atomizing the oil, and a by-pass valve for controlling the cut-off of the spray. Additions were made to the apparatus in order to record the motion of the timing valve stem photographically.

The timing valve stem was held against its seat by a helical spring so adjusted that the total hydraulic force on the stem actuated it immediately after it had

been mechanically lifted from its seat. The lift of the stem was recorded photographically to determine the effects of injection tubes 7 inches and 43 inches long. The pressure variations at the seat and in the injection valve tubes were analyzed and the lifts of the stem for both tubes computed from the analysis and compared with the experimental records.

The methods of analysis of the pressure variations and the general equation for the motion of the spring-loaded stem for the timing valve are applicable to a spring-loaded automatic injection valve, and in general to all hydraulically operated valves. A sample calculation for a spring-loaded automatic injection valve is included.

Report No. 332. The Effect of Cowling on Cylinder Temperatures and Performance of a Wright J-5 Engine. By Oscar W. Schey and Arnold E. Biermann. 22 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price fifteen cents.

This report presents the results of tests conducted by the staff of the Committee to determine the effect of different amounts and kinds of cowling on the performance and cylinder temperatures of a standard Wright J-5 engine. These tests were conducted in conjunction with drag and propeller tests in which the same cowlings were used.

The engine was mounted in the nose of a cabin fuselage and placed in the air stream of the Committee's Twenty-Foot Propeller Research Tunnel, which is located at the Langley Memorial Aëronautical Laboratory. The power was measured by means of a torque dynamometer placed within the fuselage. Sixty-nine iron-constantan thermocouples and three recording pyrometers were used for obtaining the cylinder temperature measurements.

Four different cowlings were investigated, in tests herein reported, varying from the one extreme of no cowling on the engine to the other extreme of the engine completely cowled and the cooling air flowing inside the cowling through an opening in the nose and out through an annular opening at the rear of the engine. Each cowling was tested at air speeds of approximately 60, 80, and 100 miles per hour.

For the conventional type of engine cowling the results of these tests indicate that increasing the amount of cowling has the advantage of reducing the drag, but the disadvantage of increasing the cylinder barrel temperatures. Satisfactory cooling was obtained with the conventional cowling that covered 35 per cent of the cylinder cooling area. With the conventional cowling that covered 73 per cent of the cooling area the cylinder temperatures were excessive even though a large portion of the cooling air was permitted to flow inside the cowling through slots in the front of the cowling.

For the cabin fuselage with the N.A.C.A. cowling, which completely enclosed the engine and took in all of the cooling air through a 28-inch diameter opening in the nose, the drag was reduced 40 per cent. at 100 miles per hour, as compared with the same unit with no cowling on the engine. The mean temperatures of the spark-plug boss and the cylinder head were slightly reduced for the same test conditions, but the barrel temperatures were increased.

Report No. 333. Full-Scale Turning Characteristics of the *U. S. S. Los Angeles*. By F. L. Thompson. 14 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

This paper presents a description of the method employed and results obtained in full-scale turning trials on the rigid airship *U. S. S. Los Angeles*. This investigation was requested by the Bureau of Aëronautics, Navy Department, and was carried out in conjunction with pressure distribution and stress investigations. The pressure and turning investigations were conducted by representatives of the Committee, and the stress investigation by the Bureau of Aëronautics.

The results of this investigation are not sufficiently comprehensive to permit definite conclusions as to the variation of turning characteristics with changes in speed and rudder angle. They indicate, however, that the turning radius compares favorably with that for other large airships, that the radius is independent of the speed, that the position of the point of zero yaw is nearly independent of the rudder angle and air speed, and that a theoretical relation between radius and angle of yaw in a turn gives a close approximation to actuality. The method of determining turning characteristics by recording instruments aboard the airship appears to be satisfactory, with the exception that a better method of determining the small angular velocities of airships should be devised.

Report No. 335. Aërodynamic Theory and Test of Strut Forms—II. By R. H. Smith. 41 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price twenty cents.

This report, submitted to the Committee for publication, presents the second of two studies under the same title. In this part five theoretical struts are developed from distributed sources and sinks and constructed for pressure and resistance tests in a wind tunnel. The surface pressures for symmetrical inviscid flow are computed for each strut from theory and compared with those found by experiment. The theoretical and experimental pressures are found to agree quantitatively near the bow, only qualitatively over the suction range, the experimental suctions being uniformly a little low, and not at all near the stern.

This study is the strut sequel to Fuhrmann's research on airship forms, the one being a study in two dimensions, the other in three. A comparison of results indicates that the agreement between theory and experiment is somewhat better for bodies of revolution than for cylinders when both are shaped for slight resistance. The consistent deficiency of the experimental suctions which is found in the case of struts was not found in the case of airships, for which the experimental suctions were sometimes above, sometimes below, their theoretical values.

Along with these five theoretical struts were made three empirical struts of high repute, the British strut given in Reports and Memoranda Number 183, the German strut Number 53, and the United States Navy Number 2, and all eight tested for total resistance. Of the five theoretical struts, Number I excels as a fairing, Number V as a strut. Number V and the United States Navy Number 2 have about equal merit as struts, with the German Number 53 a close second and the British a poor third, the relative merits being 100, 103, and 112, respectively, of Reynolds Number $12 \times 10^4$.

ENGINEERING TRAINING FOR OFFICER'S RANK—Report of the Council of the North East Coast Institution of Engineers and Shipbuilders, February 14, 1930. 16 pages, 8vo. Newcastle-upon-Tyne, 1930.

A number of vital questions on engineering education were raised in a paper by Sir Theodore Morrison which was read at a general meeting of the Institution on the 29th of October, 1926. The paper was entitled "How should an engineer be trained?" This report which has been prepared to discuss these questions applies to marine and mechanical engineers, shipbuilders, and naval architects. To clearly indicate the scope of the views presented, "officer's rank" in engineering is defined as the possession of a reasonably broad acquaintance with the application of scientific principles to engineering, a certain measure of general scholarship, and some practical knowledge of all the trades and professions employed in engineering works. As a counterpart of educational studies and innovations that have been carried on in this country, this examination in the field of English technical education should prove of material service for the comparison of methods applied to similar aims. The topics discussed are, training for officer's rank, psychological tests, concurrent works and college training, value of university degree, pre-university training and matriculation, pre-university training in works, period of training, schemes of practical training, and replies to Sir Theodore Morrison's questions. Three appendices amplifying these topics are included.

L. E. P.

INSTRUCTIONS FOR SERVICING DESOTO UNISTEEL BODIES, 63 pages 8vo, paper.
INSTRUCTIONS IN THE ART OF AUTOGENOUS WELDING, 18 pages 8vo, paper, Philadelphia, Edward G. Budd Manufacturing Company.

The industry of pressing sheet steel into the innumerable forms that constitute the components of automobile bodies has grown to proportions, the magnitude of which may be appreciated from the immense number of cars everywhere in evidence. The development is many-sided, considered from the standpoint of the numerous branches of machine-shop technique, all of which have made important contribution to the final result. Relatively speaking, that newcomer in machine shop practice, autogenous welding, has a place of honor in the manufacture of car bodies, for by its use these intricately fashioned steel bodies when finished become to all intent and purpose a jointless one-piece structure.

The first of these pamphlets, for the instruction of servicing plants, describes in minute detail the procedure to be employed in the replacement of parts of the several forms of body of the different models of the car above named. Numerous and thoroughly indexed illustrations accompany the text. The second pamphlet is a code of practice in catechism form for the use of operators of autogenous welding apparatus. The subject is considered under the captions, apparatus, operation, sheet-metal welding and safety hints. Relating, as they do, to the methods of this celebrated firm, these pamphlets should prove an invaluable guide in the practice of autogenous welding of sheet-steel.

LUCIEN E. PICOLET

MEMORANDA OF TOXICOLOGY—PARTLY BASED ON TANNER'S MEMORANDA OF POISONS. By Max Trumper, B.S., A.M., Ph.D., Consulting Clinical Chemist and Toxicologist. In Charge of Psycho-Biochemistry Laboratory, Graduate School, University of Pennsylvania. Formerly Lecturer on Toxicology, Jefferson Medical College, Philadelphia, Penna. Second Edition, XII—214 pages, 16mo Philadelphia, P. Blakiston's Son & Co., Inc. Price $1.50.

This "Memoranda of Toxicology" contains much which justifies its recommendation. The various types of poisons are conveniently classified under the four headings: Corrosives, Simple Irritants, Specific Irritant Poisons and Neurotic Poisons. Under Corrosives are listed the mineral and so-called organic acids as well as the strong alkalies and alkaline carbonates. Simple Irritants include several compounds of potassium, zinc, silver, tin bismuth, chromium, iron and substances of vegetable origin such as aloes, jalap, croton oil etc. Particular attention is paid to Botulism a comprehensive discussion of which is supplied by Dr. Henry Leffmann. Irritant gases and vapors as chlorine, phosgene, mustard gas and formaldehyde also are included in this division. Among the Specific Irritant Poisons are found the iodides and bromides of potassium, compounds of mercury and antimony, salts of lead, copper and barium, phosphorus, arsenic and such vegetable irritants as Cantharidies. Neurotic Poisons claim the greatest amount of space. Their number is so large and variation in specific action so diverse that it has been necessary to subclassify them under such headings as Narcotics, Anesthetics, Inebriants, Deliriants, Convulsants, Paralysants, Hyposthenisants or Syncopants, Depressants, Asphyxiants and Abortives. The Appendix includes a discourse on the bites of venomous reptiles, rabid animals and stings of insects. Mention also is made of an attempt to commit suicide through an overdose of insulin. Two tablespoonfuls of sugar making a most delectable antidote. A page on First Aid describes an universal antidote consisting of 2 parts of pulverized charcoal, 1 part tannic acid and 1 part magnesium oxide. All substances of a harmless character and which possess the ability to render harmless many other substances. The charge in an all service gas mask is described and the specific value of each constituent explained. Based upon such recent accidents as poisoning by gaseous refrigerants and by the decomposition products of burning x-ray films the author emphasises the need of special masks for protection in such instances.

This second edition has striven to keep pace with changing conditions and consequently makes mention of lead tetraethyl, poisoning by refrigerating gases such as methylchloride, asphyxiating gases as used in the war and the use of barium salts in X-ray work. The author lists the symptoms due to each specific poison, describes the most approved methods of treatments and gives careful directions for detection of and proof that the poison is present. The toxicologist should find in this book a reference work of great value and convenience. Writers of murder and mystery stories also would find much of this material to be of interest.

T. K. CLEVELAND.

ANNUAL SURVEY OF AMERICAN CHEMISTRY, Volume IV, edited by Clarence J. West, National Research Council, assisted by many contributors. 8vo, 549 pages, Chemical Catalog Company, New York, 1930. Price, $4.00.

This survey, begun a few years ago and it is to be hoped to be continued for a long while, is very valuable as it presents to American chemists a condensed picture of the work being done in all departments of chemistry in this country. The general characters of this publication have been presented in connection with previous volumes. Notice of the present volume at any length would merely repeat the general statements already made. Some data as to the contents and extent of the volume are opportune. Forty-three papers are collected, each of them a survey of the literature on a special subject of chemistry mostly in its applications, but a limited number of essays on the pure science are included. Colloid Chemistry, as might be expected, covers a considerable space. The contributors are scattered over the country as in the immediately preceding volume. There is only one article from Philadelphia, which is from the laboratory of one of the great public utilities companies. The Bibliographies given in connection with each essay show that the literature of the period included has been pretty thoroughly searched. The large amount of data collected in the book is an encouragement to American chemists, and an assurance that very great improvement is taking place in the research activities of this country. Chemists who for any reason are unable to scan the current literature will find this volume very useful, bringing as it does into compact form the reports of research in any particular field. The influence of the automobile on the rubber industry is shown by the fact that the production of crude rubber in 1928 was 658,000 tons and in 1929, 859,000 tons and even at that only about seventy per cent. of the trees were producing.

The book is neatly printed in clear type on good paper, and is certainly a valuable addition to the chemist's library, whether that of a teacher, research worker or an industrial occupation.

HENRY LEFFMANN.

ELECTROMAGNETIC PROBLEMS IN ELECTRICAL ENGINEERING. An elementary treatise on the application of the principles of electromagnetism to the theory of the magnetic field and of the magnetic forces in electrical machinery and apparatus. By B. Hague, D.Sc. (Lond.), Ph.D. (Glas.), A. C. G. I., A. M. I. E. E., Lecturer in electrical engineering in the University of Glasgow. xiv–359 pages, 8vo, cloth. New York, Oxford University Press, 1929. Price, $10.00.

As methods of procedure in the design of power apparatus become established practice, there is a disposition to rest content with theories which yield satisfactory results, even when their use entails the aid of some tentative methods in the course of the proceedure. To the end of refining the processes of design in dynamo-electric machinery and related apparatus, the author makes a searching survey of the principles of electromagnetism and the application of those principles to the solution of certain problems which arise in the operation of electric generators. The work is accordingly in two parts, the first of which deals with "general theory," the second with "applications."

In Part I, questions of general theory, conceptions regarding the magnetic

field, the definition of electromagnetic quantities, and the derivation of units are considered critically and at length. The electromagnetic field of systems of currents, principally those flowing in straight wires both in air and in the presence of iron are next considered, stress being laid upon the idea of vector potential, although attention is chiefly directed to the usual scalar potential and flux conceptions. This theoretical treatment then proceeds to the consideration of mechanical forces on linear conductors with which is a comparison of the usual method of calculating the forces on currents in a magnetic field with the ether stress theory of Maxwell. The work is of course highly mathematical and is chiefly intended for readers who are already familiar with the principles of electrical engineering and a corresponding degree of mathematical knowledge. The mathematical burden is considerably lightened, however, by the inclusion of an unusual number of steps in the derivation of formulas and an equally generous measure of explanatory matter. Part II covers a comprehensive development of the magnetic field and mechanical forces in non-salient pole machines, the field within slots and between salient poles, and other topics of like character. The work of Rogowski and of Roth on the leakage field in transformers, Strutt's solutions for the magnetic field of thick conductors with various forms of section, and an elementary treatment of the forces acting upon switchgear and other apparatus complete this unusually thorough examination of the theory of the electromagnetic field and its application.

L. E. P.

THE ALUMINUM INDUSTRY, two volumes. Volume one, Aluminum and Its Production; Volume two, Aluminum Products and Their Fabrication: both prepared by J. D. Edwards, F. C. Frary, and Z. Jeffries, with the collaboration of experts of the Aluminum Company of America. 8vo, volume one, 358 pages, volume two, 870 pages, numerous illustrations and tabular data. New York, The McGraw-Hill Book Company, 1930. Price, $12.00.

In the middle 60's of the last century, there was in the laboratory of the Central High School of Philadelphia, a cylindrical ingot of aluminum about 15 mm. wide and 75 mm. long, weighing about 30 grams, which cost above five dollars, having been imported from France as a specimen of a newly obtained metal, remarkable for several properties, especially lightness. No expectation was then entertained that the metal would ever become more than a museum specimen. Extracted by an electrical process in the days when electric currents could only be obtained by expensive and cumbersome methods, the economic problem was regarded as prohibiting cheap manufacture. The world moves, however, new methods of attacking practical problems are constantly developed and in many fields we now have vividly the truth of the remark made for an entirely different purpose, namely: "The patient search and vigil long," accomplishing the purpose. As the author remarks in the preface, the story of aluminum is not a long one. No classic writer has any allusion to it. The works of Pliny and Theophrastus are silent on the subject. The oxide, alumina, has been long known. In this word the classic writers have a say, for the Romans knew certain astringent substances, "alumen," now known to be salts of aluminum of the general nature of alum. The Greeks knew the same minerals under the title, "stypteria," from which, of course, our word styptic. It is interesting to note that Davy, who was a pioneer worker in the electrolysis of the compounds of the lighter metals, de-

cided from his experiments that in alumina there was a metal present, which he first named "alumium." Later the name became aluminium, but in 1925 the American Chemical Society definitely dropped the "i" of the last syllable and the simplified spelling "aluminum" is now, fortunately, in vogue in this country.

In the preliminary part of the book, the earlier history of the isolation of the metal is briefly presented and shows as might be expected, that it is difficult to say who really first obtained the metal in definitely recognizable form. Especially, however, is due to H. Sainte-Claire Deville the production of aluminum in fair purity at reasonable cost.

All great industries of modern times have a complex basis, that is, they are dependent upon equivalent progress in other fields. Motion pictures, for instance, depend upon the production of a flexible film, high speed emulsions, artistic treatment and so on. The enormous development of the aluminum industry obtained this upon the ability to obtain powerful constant currents of electricity, and this again largely upon the application of an economic prime-mover such as water-driven machinery. As soon as aluminum was being produced at reasonable prices, its introduction into certain industries began, but it is comparatively late that its extensive household use has developed. Aluminum kitchen ware, now so familiar, has not had an entirely free field. Two powerful interests are naturally arrayed against it; the tin plate and enamel ware industries. Of late, considerable bitterness has been shown by these interests endeavoring to discourage use of aluminum in the kitchen and the usual story is being told. Feeding experiments of various kind on animals and "poison squad" methods have been described in the journals. A peculiar fellowship is arising between the manufacturers of alum baking-powders and the manufacturers of aluminum ware, both seeking to show the entire harmlessness of aluminum compounds in the human system.

Enough of these "*obiter dicta*," "wayside speeches"; let us turn to the book in hand. It is a vast accumulation of information on the subject of a great industry of far reaching economic and social influence. One gratifying fact in regard to the metal is that its ores are apparently inexhaustible. The first volume as noted above relates entirely to the history of the production of the metal, to its properties and to the present methods of manufacture. Every point of any importance regarding this matter is treated in a clear and comprehensive manner. The second volume after a brief account of the commercial history of aluminum presents the physical chemistry of the metal, then the natural properties of the alloys, methods of producing them and the preparation of aluminum itself and its alloys for the market and the accessory applications of the compounds. Special attention is given to the problem of the employment of aluminum in the home and in food industries.

We thus have in this work a picture of the metallurgy of aluminum and the chemistry of aluminum compounds in all their relations.

HENRY LEFFMANN.

TOWARD CIVILIZATION. Edited by Charles A. Beard. Fifteen contributions on special topics by well known writers. An introduction and a summary by the Editor. 307 pages, 8vo., Longmans Green and Company, New York, 1930. Price $3.00.

The question that arises in the mind of the reviewer on taking up this book is "What is civilization?" Historians and anthropologists talk freely of earlier

"civilizations." We hear of the glory that was Greece and the grandeur that was Rome. We are told by one authority that civilization began in Egypt, by another that it began on the plains of Mesopotamia, and we hear that the island of Crete may have been very early in this field of development.

Remarkable discoveries have been made within comparatively recent times as to what is collectively known as the pre-historic man. Artistic productions have been found evidently produced by very early races, apparently devoid of all written language for many drawings contain no definite inscriptions. They are what the artist would call "proof before letters." Many years ago at a debate in an amateur literary society, one of the members, off hand, defined civilization to be the equal cultivation of the head and heart. Epigrammatical definitions are attractive but they not infrequently sacrifice completeness to conciseness. Yet this definition seems to have some value when expanded into a broad application. Certainly a development which is limited to intellectual advance and does not concern the ethical and moral improvement is hardly to be commended. We unfortunately know so little, comparatively, of the conditions of life in ancient civilizations that we cannot compare them with our own time. The last hundred years, which will include the thirty of the present century, are distinguished clearly from all predecessors by advancement in many departments of knowledge. To enumerate the valuable inventions and discoveries that have occurred in this period would be a heavy task and yet if we look at details it may be a little difficult to find just in what modern progress is principally distinguished. Surely this last hundred years is not the one in which the greatest religious development of humanity took place, that belongs to the first century. Nor is it the century of the greatest invention in the service of mankind, the art of printing, "the art preservative of all arts." Nor do we find in this modern period the greatest geographical progress. The late fifteenth and early sixteenth centuries saw the great discoveries of Columbus, Magellan and other adventurous spirits who traversed the seven seas.

It is customary to speak of the modern age as the industrial civilization. Its foundation may be considered to have been made when James Watt made the steam engine a practicable prime-mover. Many improvements in this have been made since his time, the main urge being fuel economy. Yet the world moves in cycles, and while the steam engine for many years put the water-wheel into disuse, the introduction of electric transmission has brought back water-power to an important place in industry. Whatever may be the optimistic views of writers on modern subjects, it seems impossible to overlook certain burdens which our modern industry has put upon the workers. Not enough attention has been paid to securing safety. It is true that a good deal of the difficulty lies with the workers who often neglect provisions made and advised for them. The percentage of disasters in a great industrial state such as Pennsylvania is very high and is the cause of a great deal of misery. The efforts to secure limitation of the hours of labor, restriction of the work of women and children, old age pensions, compensation for injuries, all these efforts show that our civilization has not yet solved some of the serious problems in existence.

The work before us is an essay by a number of distinguished writers to show the progress that the present industrial portion of humanity is making toward a

better life. It seems to the reviewer that the outlook is not very optimistic but at any rate conditions are still too hazy to justify positive prediction.

HENRY LEFFMANN.

## PUBLICATIONS RECEIVED.

*Annual Survey of American Chemistry.* Volume IV, July 1, 1928, to December 31, 1929. Prepared under the auspices of the Division of Chemistry and Chemical Technology, National Research Council, James E. Mills, chairman. Edited by Clarence J. West, Director, Research Information Service, National Research Council. 51 contributors. 549 pages, 12mo. New York, The Chemical Catalog Company, Inc., 1930. Price $4.00.

*The Aluminum Industry,* in two volumes. Aluminum and its production. Aluminum products and their fabrication, by Junius David Edwards, Francis C. Frary, Zay Jeffries with the collaboration of a group of experts from the staff of Aluminum Company of America. 2 volumes, illustrations, 8vo. New York, McGraw-Hill Book Company, Inc., 1930. Price $12.00.

*Toward Civilization,* edited by Charles A. Beard. 307 pages, 8vo. New York, Longmans, Green and Company, 1930. Price $3.00.

*Gravitation, Lumière et Électromagnétisme* (*Synthèse physique*), par Emile Sevin, preface de M. Maurice d'Ocagne. 61 pages, illustrations, 8vo. Paris, Librairie Scientifique Albert Blanchard, 1930. Price 18 francs.

*The Structure of Line Spectra,* by Linus Pauling and Samuel Goudsmit. 263 pages, illustrations, 8vo. New York, McGraw-Hill Book Company, 1930. Price $3.50.

*National Research Council of Canada.* On rectification and detection, rectification and power pack design by Richard Ruedy. Pages 101–130, illustrations, 8vo. Reprinted from the Canadian Journal of Research, 2: 1930.

*Radio Data Charts.* R. T. Beatty, D.Sc. 82 pages, illustrations, quarto. London "The Wireless World," Iliffe and Sons, Ltd., 1930. Price 4 shillings 6 pence net.

*Steel Treating,* Books of interest to members of the American Society for Steel Treating. Introduction by Beatrice Winser, Librarian. 16 pages, 24mo. Newark, N. J., Public Library, 1930. Price 15 cents.

## CURRENT TOPICS.

**Lightning.** G. C. Simpson. (*Nature*, Nov. 23, 1929.) In this, the twentieth Kelvin Lecture before the Institution of Electrical Engineers, the author maintains the startling thesis that in an electrical storm the clouds are insulators surrounded by a conductor, the air. He says "A cloud is one of the best insulators we have. In fact, I myself have exposed charged bodies within clouds and been unable to observe the least loss of charge in an hour or more." As to the electrical conductivity of air, in 1887 Linss found by experiment that a charge leaks faster from an electrified body in fine weather than in damp air. The process by which air conducts is now fairly well understood. A gas molecule loses an electron and thus becomes a positive ion. Another gas molecule captures an electron and thereby is turned into a negative ion. "In the lower atmosphere there are generally about 500 ions of each sign in a cubic centimetre. They move with a velocity of about 1½ cm. per sec. in a field of 1 volt per cm." As a result of the presence of these mobile ions the resistivity of the lower atmosphere amounts to $4.5 \times 10^{15}$ ohms in clear weather. To throw light upon the significance of this value, Swann is quoted as saying "A column of air one inch long offers as much resistance to the passage of the electrical current as a copper cable 30,000 million million miles long, and of the same cross-section, i.e. as much resistance as that of a copper cable long enough to reach from here to Arcturus and back 20 times."

The theory of a discharge of lightning is discussed in detail. It is concluded that the branches of the stroke point away from the seat of the positive charge. A laboratory experiment confirms this. On a photographic plate were placed two copper discs. These were joined to the terminals of a Wimshurst machine. The resulting discharge was registered on the plate. It showed long thin sparks spreading out as they left the positively charged disc. At the negative disc there was but little shown.

Simpson cites with approval C. T. R. Wilson's estimates of the quantity of electricity concerned in a lightning flash as from 10 to 50 coulombs and of the potential before the flash as of the order of a billion volts. The energy of a 20-coulomb flash amounts to $10^{17}$ ergs or about 3,000 kilowatt-hours. By no means all of this appears at the surface of the earth, much being dissipated along the path Recent observations from several investigators go to show that a discharge lasts on the average somewhat more than .001 sec. 20

amperes in .001 sec. means a current of 20,000 amperes with perhaps an instantaneous value reaching 100,000 amperes.

A drop of water falling through still air does not go on increasing continuously in velocity but owing to the retardation of the air attains a steady velocity that is dependent on its size. Lenard in Germany measured these end velocities and found them to be as follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Diameter in cm..... | .100 | .200 | .300 | .350 | .450 | .546 | .636 |
| End Velocity in cm. per sec..... | 440 | 590 | 690 | 737 | 805 | 798 | 780 |

The maximum velocity observed was about 800 cm. per sec. The decrease in velocity for drops larger than .45 cm. in diameter is attributed to their flattening which makes them encounter increased resistance from the air. This causes them to break up into smaller drops. Simpson in India in 1908 discovered that there is a separation of electricity when a drop of water breaks without impact against a solid. "Every time a drop breaks, the water of which the drop is composed receives a positive charge. The corresponding negative charge is given to the air." This portion of air with its negative charge is swept away by currents while the positive charge grows larger by the breaking of additional drops, but escapes only with difficulty from the region where the disruption of the drops occurs. Thus there is a separation of the positive and negative charges and a building up of the potential difference.

G. F. S.

**Iodine Research Program.**—Since January 1, 1928, Mellon Institute of Industrial Research, Pittsburgh, Pa., has had in operation a Multiple Industrial Fellowship founded for the purpose of investigating the properties and uses of iodine. This Fellowship, which is sustained by the Iodine Educational Bureau, 64 Water Street, New York, N. Y., is headed by Dr. George M. Karns, formerly a member of the chemical faculty of the University of Illinois. All results of the Fellowship studies will be published.

Recently, through an additional appropriation from the Fellowship donor, Mellon Institute, acting for the Iodine Fellowship, has made arrangements for the study of certain iodine problems in other institutions that have special facilities for such types of work. On October 7, 1929, a scholarship was founded at the Philadelphia College of Pharmacy and Science by a research grant from the Institute. This scholarship—which, for the college year 1929–30, will be held by Mr. L. F. Tice—will have for its aim a broad investigation of vehicles and solvents for iodine, especially for external use in medicine. A large number of new organic chemicals will be

studied as solvents with the object of evolving, if possible, a more satisfactory preparation than the alcoholic tincture now in use. The research, for which a definite program has been laid down, will be supervised by Professor Charles H. LaWall, Dean, with the advisory collaboration of other faculty members of the Philadelphia College of Pharmacy and Science and with the direct coöperation of Dr. Karns. The investigational findings of the scholarship will be reported in the literature.

Another phase of the research program includes a grant made on September 26, 1929, to the Pennsylvania State College for a comprehensive investigation—under the direction of Professor E. B. Forbes of the Institute of Animal Nutrition—of the nutritional place and value of iodine in the feeding of live stock. Despite the large amount of work which has been done on the rôle of iodine in metabolism, especially with reference to the thyroid, very little is known regarding the specific dietetic aspects of this element, particularly in the lower animals. Dr. Karns and his coworkers on the Iodine Fellowship of Mellon Institute are coöperating closely with Dr. Forbes and his staff, mainly by preparing standardized feeds. The findings of this research also will be made available to the public, in accordance with the Iodine Educational Bureau's policy of disseminating to every one interested the results of all investigations made under its ægis.

Mellon Institute is giving consideration to the founding of a research scholarship in a medical school for the purpose of aiding in the solution of incompletely answered questions respecting the utility of iodine in internal medicine. A number of pharmacologists are aiding the Institute in determining a program for such pharmacodynamic inquiry.

R.

**Some Factors Governing the Magnitude of Frictional Electric Charges.** P. A. MAINSTONE. (*Phil. Mag.*, Nov., 1929.) Macky in 1928 published the results of his investigations on the production of electricity by friction in which for the first time the effects of changes of pressure in the surrounding gas were observed. He noted that for a given rubbing process the quantity of electricity developed diminished as pressure was reduced and at a pressure of 1 or 2 mm. it became almost zero. In the present piece of work the effects of changing pressure are carefully observed.

With a preliminary apparatus certain interesting and suggestive facts were brought out. Potentials ranging from 10 to 20 volts were developed even when the amount of work done in friction was very small. The presence of water vapor greatly reduced the charge

developed. "The charge per single rub was generally found to increase with the number of previous rubs until a steady value was reached." This effect, found previously by several others, is attributed to adsorbed gas on the rubbing surfaces.

The final form of friction chamber was a vertical brass cylinder 10 cm. long and 8 cm. in diameter, closed at both ends by discs of the same metal. An opening in the lid led to the pressure pump, while through a second aperture in it projected a glass taper joint carrying at its lower end a horizontal arm at the farther extremity of which was attached one of the substances to be rubbed. Two pieces of the other substance with exactly similar free upper surfaces on the same level were mounted on the base of the cylinder. By rotating the tapering piece in the joint the substance on the arm was made to sweep over one piece of the second substance, insulated and attached to an electroscope, and to pass without jolt to the second piece which was earthed. Pressure within the cylinder was reduced by steps, measurements of the charge developed by a rub being taken at each pressure.

Whether in air, nitrogen or hydrogen, the potential developed by rubbing glass on brass followed this course with change of pressure. As pressures were made lower than atmospheric the potential grew less slowly, but fell quite rapidly from 40 to 10 cm. At 10 cm. the potential was a minimum having a value about 20 per cent. as large as at atmospheric pressure. As the pressure was further reduced the potential scarcely changed until at 1 mm. a sharp rise occurs which persists up to the smallest pressure used, about .01 mm. At this small pressure the potential was somewhat greater than at 760 mm. Quite similar are the results of rubbing glass on steel in air, nitrogen and hydrogen. For pressures extending from atmospheric down to about 10 mm. the potential varies with the surrounding gas, being greatest with air and least with hydrogen, but at very low pressures the nature of the gas makes little or no difference. This is explained by the author as due to adsorption of gas which modifies the character of the rubbing surfaces. "At very low pressures the amount of gas adsorbed becomes negligibly small, and when this stage is reached the contact between the glass and the metal will become the more important factor. As the gas pressure is reduced to the lowest available limit, the glass-metal contact becomes more perfect, with a corresponding increase of charge."

G. F. S.

**The Change of Electrical Conductivity in Strong Magnetic Fields.** P. Kapitza. (*Proc. Roy. Soc.*, A791.) The author has a remarkable electromagnet by which fields up to 300,000 gauss can

be produced for a short time. Using it he studied the change of resistance of bismuth which has long been known to vary with the strength of the magnetic field. "In agreement with previous investigators it was found that the resistance of bismuth in weak fields increases in proportion to the square of the magnetic field, and in stronger fields follows a linear law, the increase of resistance being proportional to the magnetic field up to fields of 300 kilogauss. It was further found that this linear part of the change of resistance is, in most cases, independent of the orientation of the crystal in the magnetic field, and also of the degree of perfection of the crystal. This suggests that we are concerned with an atomic phenomenon." Other substances were discovered to act as bismuth does in that they follow first a square law and then a linear law in fields exceeding 60 or 100 kilogauss.

The resistance specimen of the element under examination was contained in a Dewar flask located between the poles of the electromagnet. The flask contained liquid nitrogen or solid $CO_2$ in ether. Both the current through the resistance and the potential across its terminals were recorded by oscillographs. From these curves the resistance could be calculated. When the element could be made into wire it was on an ebonite cylinder. For liquids like mercury and gallium a thread cut on ebonite was filled with the liquid. Chromium, beryllium, arsenic, etc., though solid do not furnish flexible wire. These were used in the form of short rods only 5 or 6 mm. long with very small resistance. Precautions had to be taken to prevent any motion of the resistance in the flask, because any displacement of it set up an induced electromotive force superimposed on the difference of potential being measured.

The presence of impurities in the elements greatly affected the results, always reducing the change of resistance. "To ensure that the impurities shall have only a small influence on our results the substance must in no case be less than 99.9 per cent. pure, and must be even purer in the case of the lighter elements." To get uncontaminated wire one lot was drawn to clean the sapphire die, then a second lot was drawn for the experiment.

Most elements were studied at three temperatures, at room temperature of about 290° K., at about 193° K. when the Dewar flask held solid $CO_2$ and ether, and at 88° K. with the flask holding liquid nitrogen. Liquid air had to be discarded because it was so strongly paramagnetic that it was moved by the magnetic field.

The change of resistance due to the magnetic field transverse to the coil or rod has been measured for the following, (*a*) First Group of elements, Li, Na, K, Pb, Cs, and Cu, Ag, Au. (*b*) Second Group, Be, Mg, Zn, Cd, Hg. (*c*) Third Group, Al, Ga, In, Tl.

(*d*) Graphite, Ge, Sn, Pb, Zr, Th. (*e*) Fifth Group, As, Bi, Sb, V, Ta. (*f*) Sixth Group, Cr, Mo, W, Te. (*g*) Seventh Group, Mn. (*h*) Eighth Group, Fe, Ni, Pd, Pt. Nickel was the only element found for which the resistance grew less as the strength of field was increased. For nearly all the others, as the field was made larger, the fraction obtained by dividing the increase of resistance due to the field by the resistance at the temperature of the experiment in the absence of any field was at first proportional to the square of the field but for larger values of the field it became proportional to the field simply. The author derives formula expressing these relations in both of which a factor of proportionality appears, $\beta$. In all the groups of the periodic system it is noticed that the value of $\beta$ decreases as we pass from the lightest to the heaviest element, except in the Fifth Group where bismuth at the temperature of liquid air has a value of $\beta$ equalling 450° whereas arsenic and antimony, the lighter elements, present values of 123 and 179 respectively. Within the same group the range of values for $\beta$ may be considerable. In the Second Group at liquid nitrogen temperature beryllium has $\beta$ equal to 10.3 and mercury to .5 (all the values of $\beta$ quoted are to be divided by 1,000,000). In general $\beta$ becomes greater with lowering of the temperature. For instance the values for bismuth at room, solid $CO_2$-and-ether and at liquid air temperatures are 120,680 and 4500. G. F. S.

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